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ABSTRACT

This guide contains six sections which offer suggested sequential steps for teaching addition, subtraction, multiplication, division, fraction concepts and algorithms, and decimal fractions. Based on the manipulative approach, the guide offers activities and teaching methods which give students many opportunities to work with models. This approach gives the students, who are in the concrete stage of development as defined by Piaget, help with the discovery and understanding of the mathematical concepts. To help the teacher guide the students through the lessons, a two-column format is used in which sample questions and teaching suggestions are listed to the right of the activities. An introductory page for each section describes the approach used and presents background information on children's learning and on the mathematical operation or concept involved. Des Moines (Iowa) Mathematics Performance Objective numbers are listed to the left of appropriate lessons. (DC)

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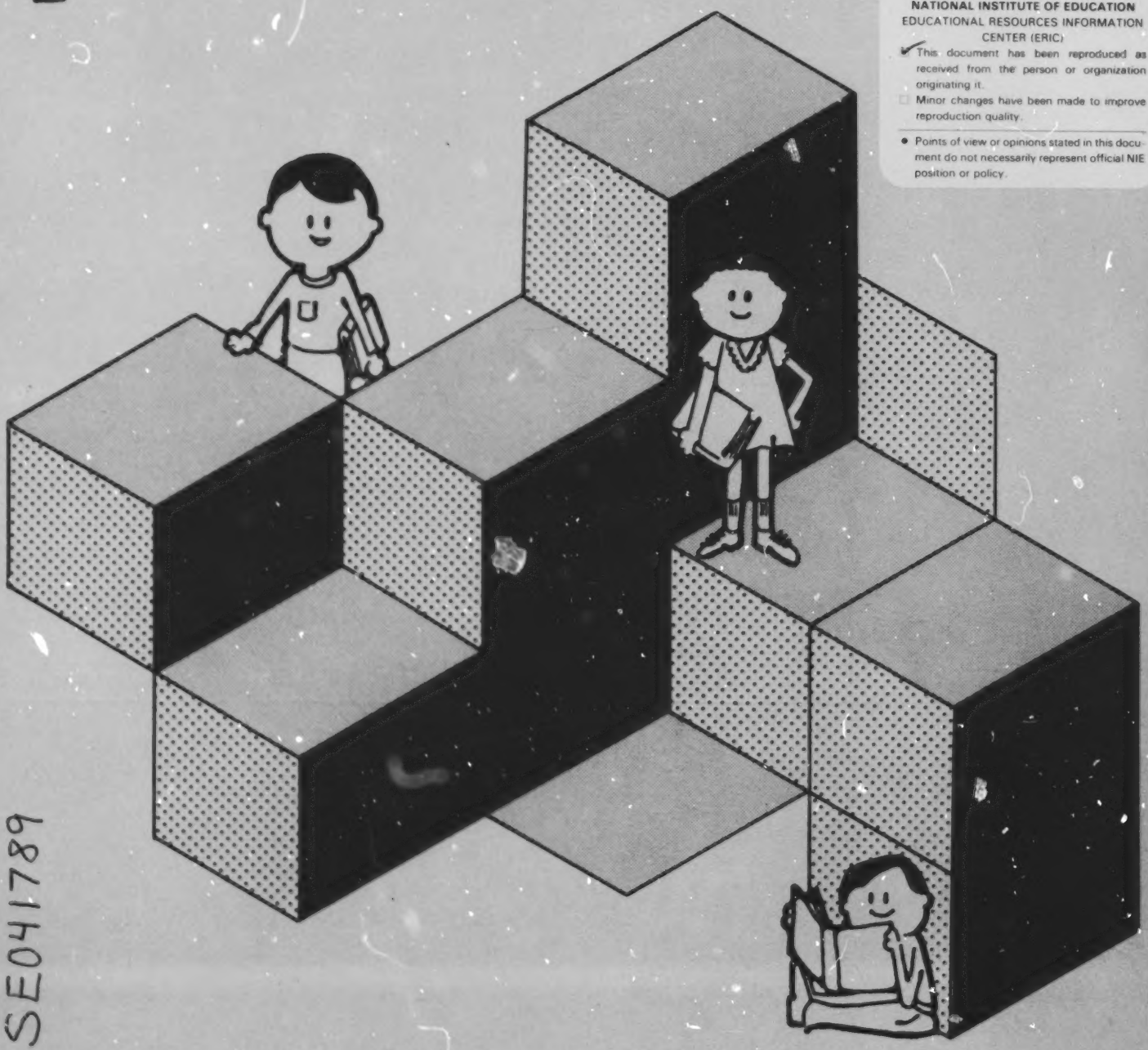

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Title 1 Mathematics Des Moines Public Schools

DES MOINES INDEPENDENT COMMUNITY SCHOOL DISTRICT
1800 Grand Avenue
Des Moines, Iowa 50307

Teaching Strategies
Title I Elementary Mathematics Program

1978-79

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ADDITION

Addition of whole numbers is approached through the concept of the union of two disjoint sets. Children can profit from the following three types of experiences to help them learn the basic facts and understand addition.

(1) Activities with concrete materials to develop their concept for the operation and to develop the appropriate language and symbolism

(2) Thinking strategies to help them organize and understand the relationship among facts and to make recall easier

(3) Drill to help them increase speed and accuracy

The following guide is a series of suggested, sequential steps for teaching addition. The steps deal with the concrete stage of development and with teaching strategies. It is based on the manipulative approach, giving the child many opportunities to work with models. These models help the child with the discovery and understanding of the addition concepts. The teacher should guide the student through the activities by asking appropriate questions. Throughout the guide, sample questions and suggestions are listed in the column to the right of the lessons. When they correspond, Des Moines Mathematics Performance Objective numbers are listed to the left.

Beginning level

Develop need from real-life situations:

Example: "How many girls are in your family? How many boys? How many children are in your family altogether?"

(Story situations need to be presented frequently throughout the entire sequence of teaching steps.)

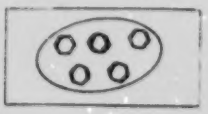
Note: A mat (construction paper, felt board, place-mat, etc.) is used to distinguish and separate the objects used in the problem situation from other extraneous objects. This device helps avoid confusion in the child's mind.

A-1 Joining sets:

Form 2 sets of objects on a mat

Count the number of the first set. Count the number of the second set.

Join the sets (push them together). Count the total number.



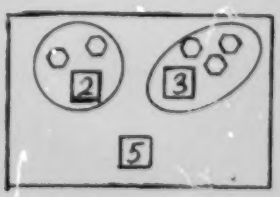
Story: "I have 2 cookies. John gave me 3 more. How many do I have now?"

Encourage student verbal response: "I have 2 cookies. John gave me 3 more. I have 5 cookies now."

Note: Some children will recognize the total number without counting.

Numerals to correspond with sets:

Form 2 sets of objects. Select the numeral for each set and place it with the set. Select the numeral for the sum.

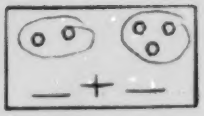


(Child has a set of numeral cards 1 through 9)

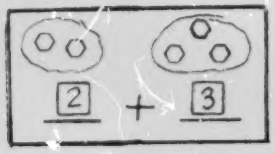
"How many cookies did you have? How many did John give you? How many do you have now?"

The + sign:

Form 2 sets.



Select the numerals and place them in the frame.

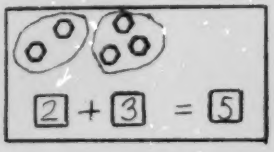


"This is the number phrase we write to explain our action. We read it 'two plus three.' We add the number 3 to the number 2.

Note; The + sign is not properly used between sets, but rather between numerals.

The = sign:

Select the numerals for the sets and the sum.



"2 + 3 and 5 name the same number. 2 + 3 is the same as 5. We read it, "two plus three equals five."

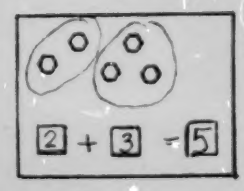
Note: Introduce the "counting on" strategy here. (See strategy section, page 6.)

A-13 The commutative property (a + b = b + a):

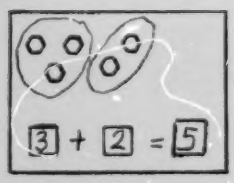
Show that when adding two numbers, the order of the addends does not affect the sum.

Form 2 sets. Select the numerals. Change the places of the sets and numerals.

"When we join a set of two with a set of three, we have 5. When we join a set of three with a set of two, what do we have?"



Step 1



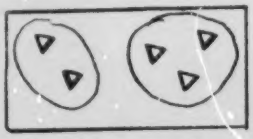
Step 2

A-3 Missing addends:

Form a set of < 9 objects. Select 1 to 8 objects to put with the first set to make a total of ≤ 10.

2 objects and ____ objects are 5 objects.

Write the equation with the sets.

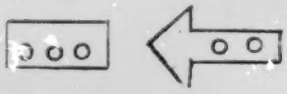


2 + 3 = 5

"Make a set of two objects. How many will we join with two objects so that we have 5 altogether?"

A-5
A-7
B-15 The vertical form:

Use objects to show the joining of sets. Write the horizontal and vertical form.



3 + 2 = 5

$$\begin{array}{r} 3 \\ +\,2 \\ \hline 5 \end{array}$$

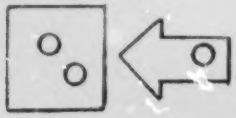
(Give the student practice in writing and reading both forms)

"This vertical form is another way to record the action. It is the computational form. We read it, 'three plus two is five'."

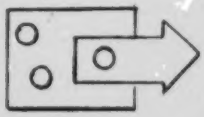
(The vertical form is not a number sentence.)

Inverse relationship.

Using objects, show the addition action and corresponding subtraction action for that set.



2 + 1 = 3

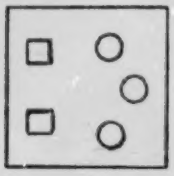


3 - 1 = 2

"What kind of action took place in the first sentence?"
(Joining, which illustrates addition)
"What sign (operation, symbol) did we use in the first number sentence?"
(The plus sign)
"What kind of action took place in the second sentence?"
(Separating sets, which shows subtraction)
"What sign did we use in the second number sentence?"
(The minus sign)

B-19 Family of facts:

Complete the four sentences that belong to a family of facts. Example: Show objects clearly separated into sets of 2 and 3.



2 + 3 = 5
3 + 2 = 5
5 - 3 = 2
5 - 2 = 3

Note: Be sure to teach numeration objectives A-3 through A-6 on teen numbers before teaching addition facts for sums 11 through 19.

The facts 10 through 19:

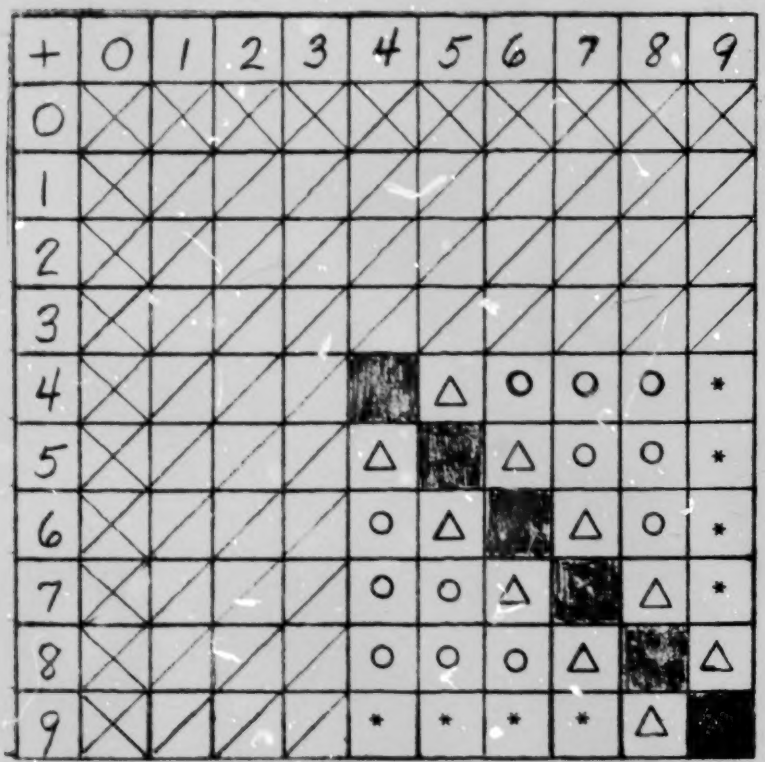
Use steps similar to the preceding suggestions.

Strategies for addition:

The next few steps are strategies for helping children see the relationships among facts.

Develop an Addition Table

Teacher leads and records students' thinking. Then the students make their own charts using counters if necessary.



A table makes it apparent that there are 100 and only 100 permutations (arrangements) of the addends. Show that the facts on the bottom "half" of the chart are the commutative form of the top "half".

(Commutative property of addition: (a + b = b + a)

Zeros - After studying the 0 examples, students can discover that in each example one addend (a zero) is common to all. The sum is the non-zero addend. Children are able to generalize that adding zero to another number doesn't change the non-zero numbers. This is called the identity element of additon.

10 facts and 9 commutative forms (19)

Counting on - This technique is most efficiently used when adding 1, 2, or 3 more. It is difficult for children to count on more than 3 because it is hard to remember both what comes next and how many

24 facts and 21 commutative forms (45)

have been counted. Children should start with the larger addend.

Doubles (4's to 9's) - Children are generally able to learn the doubles easily. Counting by 2's can help.

6 facts

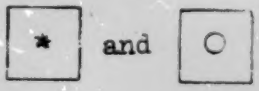
Associative property -

One more, one less - Show that the near doubles can be derived by adding 1 to, or subtracting 1 from, the doubles.

5 facts and their commutative forms (10)

Sharing - Since children have already been developing a sense of tenness in place value, the facts of 10 can be used to help with others. Both addends are changed; one increases as the other decreases. The student thinks,

10 facts and their commutative forms (20)



9 + 6 is
9 + (1 + 5)
(9 + 1) + 5
10 + 5
15

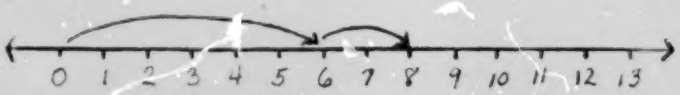
Note: Facts designated on the addition table are easier than facts designated This is very difficult thinking for some children. Some will probably need to memorize these facts by rote.

8 + 5 is
8 + (2 + 3)
(8 + 2) + 3
10 + 5
15

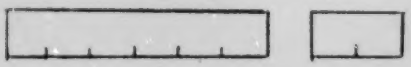
Note: More than one strategy can be used for many facts.

B-17 Numberline or rods:

These are models which can be used to increase understanding of addition.



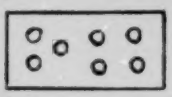
$6 + 2 = \underline{8}$



$6 + 2 = \underline{8}$

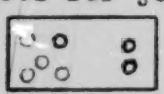
B-19 Facts grouped by sum:

Lay out a set of objects.



7

Move these objects into 2 sets. Record the addition facts for joining them.

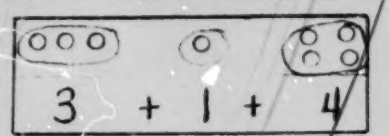


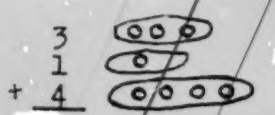
5 + 2

Allow children to discover the various combinations of the first set. Record these.

5 + 2	2 + 5
6 + 1	1 + 6
4 + 3	3 + 4
7 + 0	0 + 7

B-18 Add three 1-digit numbers:





Show studerts that any two numbers could be added first. Have students look for 2 addends for which they know the sum; then combine with third addend.

1

B-23 Associative property (with parentheses):

$3 + (1 + 4) =$
$3 + \quad 5 \quad = 8$

$(3 + 1) + 4 =$
$\quad 4 \quad + 4 = 8$

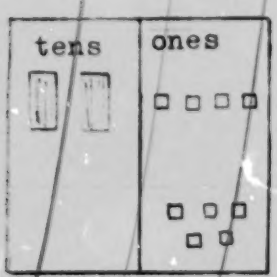
Explain that in math, if there is more than one joining of sets, the parentheses show which two numbers to add first.

Note: Manipulation of objects to form models of teen numbers (Numeration Objective B-20) should precede the next step.

B-15a 2-Digit plus 1-digit (no renaming):

Present a story situation. Example: There are 24 children playing on the playground. 5 more came to join them. How many children were on the playground then?

Lay out twenty-four objects. (Use metric/Dienes blocks, ice cream sticks, beans, etc.) Lay out the 5 objects.



	tens	ones
	2	4
+		5

The larger numeral should be the second addend at times so the student will see both forms.

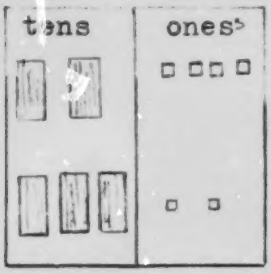
$$\begin{array}{r} 5 \\ +\ 24 \end{array}$$

Push the ones together and the tens together. Record the action and the sum.

After working with objects, move to the pictured, then the abstract stages.

B-29 2-Digit plus 2-digit (no renaming):

Follow steps as outlined above in moving from concrete to abstract stages.

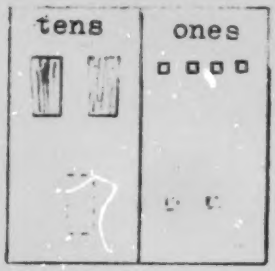


	tens	ones
	2	4
+	3	2

B-34 Missing addends with 2-digit numbers (no renaming):

Present a story situation. Example: There are 24 children here for the play. We want to use 36 children in the play. How many more children will we need?

Lay out 24 blocks. Put more on the mat so that you have 36. How many did you put on?



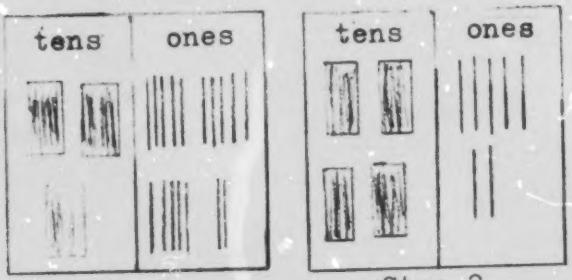
	tens	ones
	2	4
+	1	2
	3	6

Note: Various manipulatives should be used at the concrete stage. Move from separate objects (ice cream sticks or Unifix cubes) to bean sticks or metric/Dienes blocks, then to money. Poker chips (where each chip equals a place value -- i.e., 1 blue chip = 100, 1 red chip = 10, and 1 white chip = 1) could be used at a later stage.

Introduction to renaming:

Lay out 3 tens bundles and 17 ones. Record this. Using the ones, make 1 more tens bundle. Place the new tens bundle in the tens place and record.

"Trade 10 ones for 1 ten."



Step 1 Step 2

3 tens 17 ones =

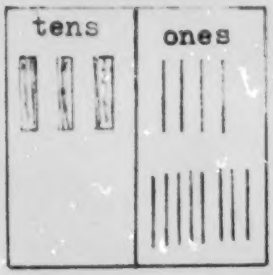
4 tens 7 ones

C-46 2-Digit plus 1-digit with renaming:

Present a story situation. Example: There are 34 children in the class. 8 new students enrolled. How many children were in the class then?

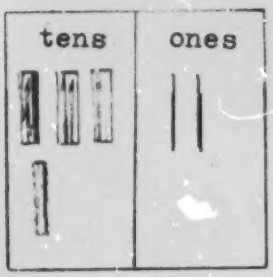
Lay out 34. Lay out 8 more. Push the ones together. Push the tens together. Since there are enough ones to make a set of 10, move it to the 10's place. Record the action.

"Trading again."



Step 1

$$\begin{array}{r} 34 \\ +\ 8 \\ \hline \end{array}$$

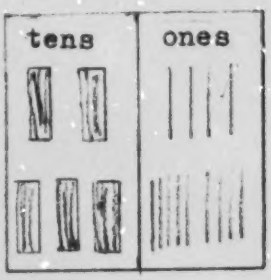


Step 2

$$\begin{array}{r} {\scriptstyle 1}\,34 \\ +\ 8 \\ \hline 42 \end{array}$$

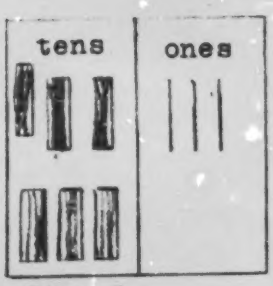
C-48 2-Digit plus 2-digit with renaming, sums < 100:

Follow steps as previously outlined in moving from concrete to abstract stages.



Step 1

$$\begin{array}{r} 24 \\ +39 \\ \hline \end{array}$$



Step 2

$$\begin{array}{r} {\scriptstyle 1}\,24 \\ +\ 39 \\ \hline 63 \end{array}$$

Note: Children sometimes benefit from learning a 2 step method of recording.

$$\begin{array}{r} 24 \\ +\ 39 \\ \hline 13 \\ 50 \\ \hline 63 \end{array}$$

Some children will be able to do the renaming without recording it.

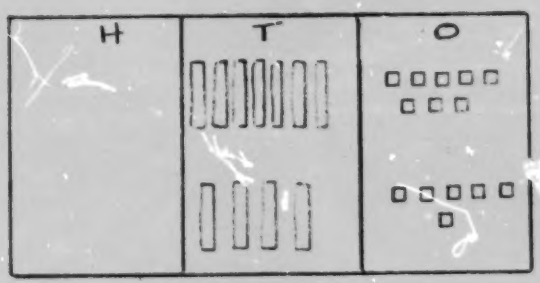
$$\begin{array}{r} 24 \\ +\ 39 \\ \hline 63 \end{array}$$

C-49 Add three 2-digit numbers, sums ≤ 100:

$$\begin{array}{r} 17 \\ 24 \\ +\ 35 \\ \hline 76 \end{array}$$

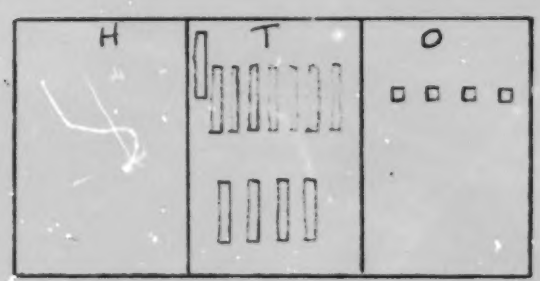
> Note: Be sure to teach the numeration objectives C-42 through 54 before the next step.

C-50 2-Digit plus 2-digit with renaming, sums > 100:



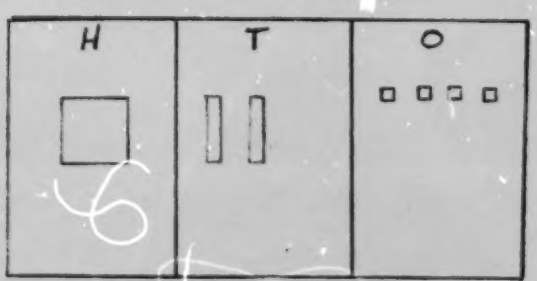
Step 1

$$\begin{array}{r} 78 \\ +\ 46 \\ \hline \end{array}$$



Step 2

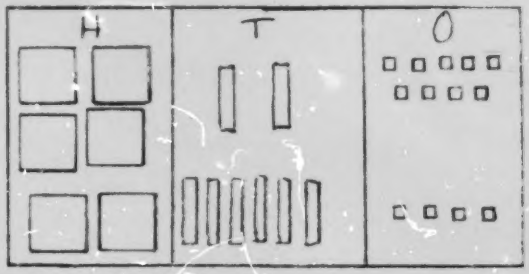
$$\begin{array}{r} {\scriptstyle 1}\ \ \\ 78 \\ +\ 46 \\ \hline 4 \end{array}$$



Step 3

$$\begin{array}{r} {\scriptstyle 1}\ \ \\ 78 \\ +\ 46 \\ \hline 124 \end{array}$$

C-54 3-Digit plus 3-digit, renaming ones:

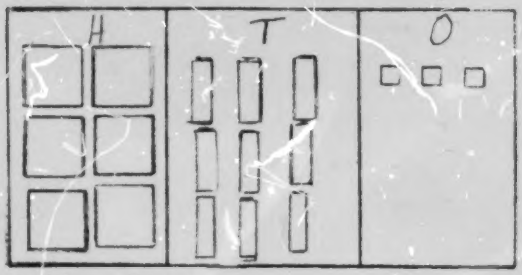


Step 1

429
+264

Note: Give students related examples, such as:

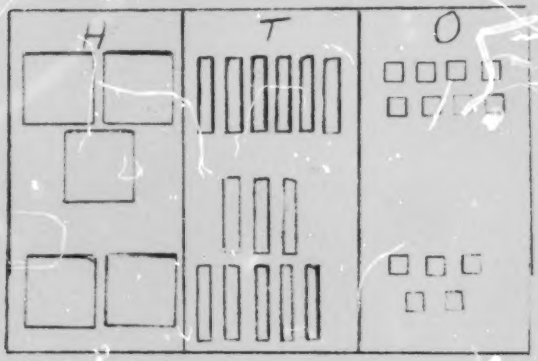
429 + 4 = 429 + 64



Step 2

429
+264
693

C-54 3-Digit plus 3-digit, renaming ones and tens, sums < 1000:

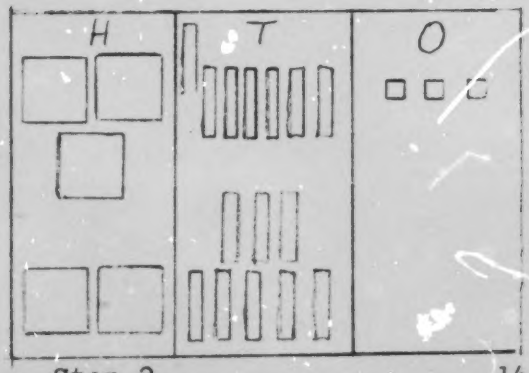


Step 1

368
+285

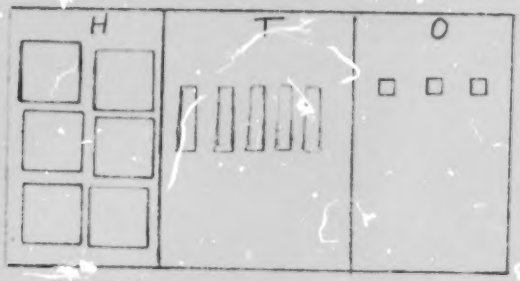
Note: Give students related examples, such as:

378 + 64 = 479 + 64 = 462 + 381



Step 2

368
+285
653



Step 3

368
+285
653

C-55 3-Digit plus 3-digit, renaming, sums ≥ 1000:

246	286	289
+ 942	+ 942	+ 942

Note: At these levels, students will move more quickly through the steps from manipulative to abstract. Remember, we must use models to promote understanding, but our purpose is to develop the algorithm - - - with understanding. Therefore, don't overuse the model.

C-56 Add any three 3-digit numbers:

237	481	721	865
246	292	832	998
+ 109	+ 153	+ 603	+ 537

C-57 Add any two 4-digit numbers:

4,847	2,187	2,859
+ 3,059	+ 7,643	+ 4,286

D-86 Add any 2 to 5 numbers, ≤ 10,000:

9,087	8,765	1,234
+ 4,762	3,044	5,678
	+ 4,928	9,012
		3,456
		+ 7,890

E-89 Add any 2 to 10 numbers, each ≤ 1,000,000:

987,654
123,456
54,321
708,090
361,172
+ 121,212

SUBTRACTION

The solution of subtraction situations can be approached in several ways: (a) separating objects from a given set; (b) comparing the numbers of two different sets. This paper addresses itself to (a); (b) will be developed in another paper.

Three types of experiences help the child understand subtraction and master facts :

(1) Concrete stage: real life situations and familiar objects are presented to develop understanding and the appropriate language and symbolism.

(2) Representational stage: pictures used in place of concrete objects

(3) Abstract (symbolism) stage: drill/computational activities to develop immediate recall and accuracy

This guide is a series of suggested sequential steps for teaching subtraction. The steps deal with the concrete stage of development and with teaching strategies. It is based on the manipulative approach, giving the child many opportunities to work with models. These models help the child with the discovery and understanding of the subtraction concepts. The teacher should guide the student through the activities by asking appropriate questions. Throughout the guide, sample questions and suggestions are listed in the column to the right of the lessons. When they correspond, the Des Moines Math (K-8) Performance Objective numbers are listed to the left.

Beginning level

Develop need from real-life situations:

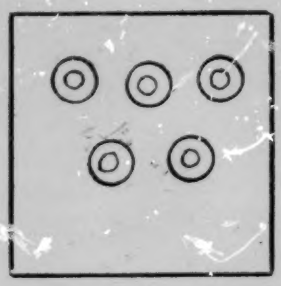
Example: "There were 4 children in our group. 1 child left the group. How many children are left here?"

(Story experiences should be presented at all stages to relate math problems to everyday experiences.)

Note: A mat (construction paper, felt board, placemat, etc.) is used to distinguish and separate the objects used in the problem situation from other extraneous objects. This device helps avoid confusion in the child's mind.

A-2 Separate sets:

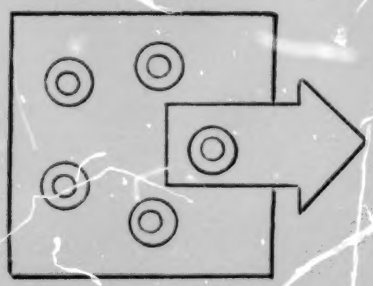
Form one set of objects on a mat.



The concept of subtraction is introduced in the initial stages by using the idea of "take away" or separation.

"Take away" is properly used when separating objects from a given set and observing how many objects remain.

Separate the set.



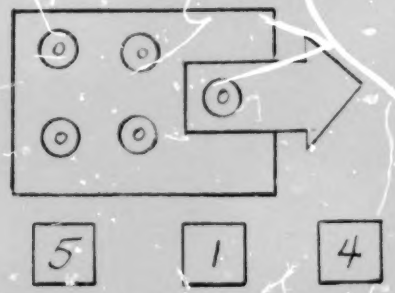
Story problem: "I had 5 lifesavers. I ate 1. How many lifesavers do I have left?"

Questions to prompt child's thinking: "How many lifesavers did you have at first? How many did you eat? How many do you have left?"

Encourage this thinking and verbal response: "I had 5 lifesavers. I ate 1. Now I have 4 lifesavers."

(Teacher may record these numerals on board.)

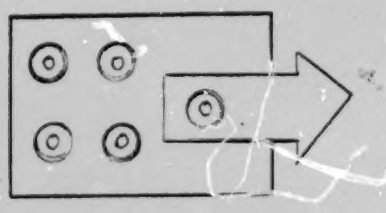
Numerals to correspond with sets:



(Child has set of cards 1 through 9)

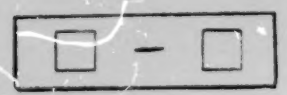
"How many lifesavers did you have?" Child places numeral 5. "How many did you eat?" Child places numeral 1. "How many lifesavers are left?" Child places numeral 4.

The minus or subtraction (-) sign:

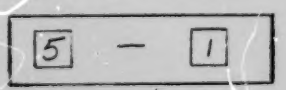


Provide frame with symbol and space for numeral.

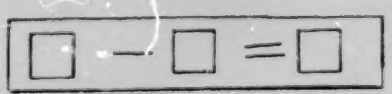
This frame will hold the number phrase which shows the action. The phrase is read "five minus one." We subtract one from five.



Select the numeral and place in frame.



Review the = sign in forming a complete subtraction sentence:



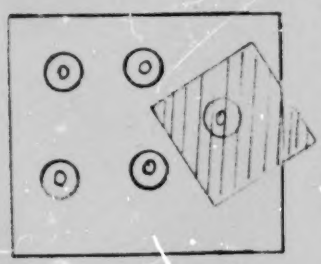
Give child number sentence form with blanks for numerals and symbols in appropriate places. Child manipulates objects and selects corresponding numerals to place on his number frame. Teacher reads completed number sentence "5 minus 1 equals 4". Encourage child to read his number sentence aloud, saying a word for each symbol.

Give students opportunity to form many different number sentences.

Note: After using the concrete steps suggested, picture or corresponding workbook pages could follow each step as child demonstrates readiness.

At this stage the child should have many experiences writing number sentences to correspond to objects, pictures and oral story problems.

A-4 Missing addends in subtraction:



Show a set.

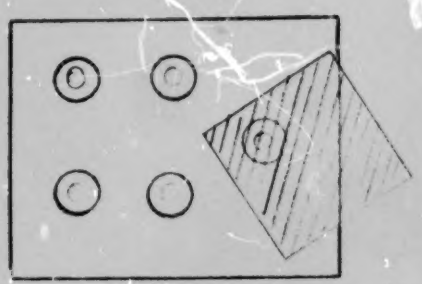
Cover part of set (or take away).

How many are left?
How many are covered?

Help children record known facts in an effort to find the missing facts.

Note: Another way of finding the difference is adding on. "What must be added to 4 to get 5?"

Continue with missing addends in subtraction using a number sentence board.



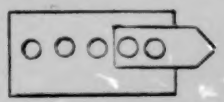
$\boxed{5} - \square = \boxed{4}$

Suggested activity: Label small box with numeral and put corresponding number of objects in box. Ask child to count objects in box. Remove some objects from box. Give box to child for him to count number of objects in box.
Ask child to place in the sentence board the numeral for number of objects in original set: $\boxed{5}$. Place numeral for number left in the set: $\boxed{4}$. "These are the known facts." Help child discover how many objects are covered or missing: $\boxed{1}$.

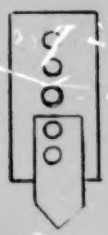
A-6
A-8

Vertical forms for subtraction:

Demonstrate with objects to illustrate that placing objects in another pattern does not change facts. Give children practice in writing and reading number sentences in both forms.



5 - 2 = 3



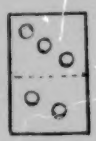
$\begin{array}{r} 5 \\ -2 \\ \hline 3 \end{array}$

"Another way to record the action is

$\begin{array}{r} 5 \\ -2 \\ \hline 3 \end{array}$

This is the computational form and is read "five minus two is three".

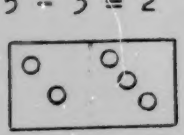
Domino cards are useful here.

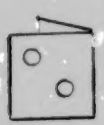


$\begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array}$

Fold card

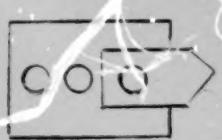
5 - 3 = 2



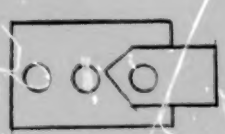


Inverse relationship:

Show the corresponding addition fact related to subtraction using objects. Child manipulates objects on mat.



3 - 1 = 2

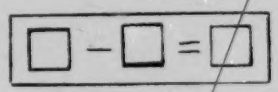


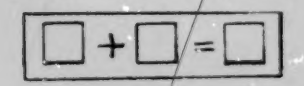
2 + 1 = 3

"What kind of action took place first? What sign is used?"

"What kind of action took place in second example? What sign is used?"

Ask child to choose corresponding numerals to place on his number frame.





B-15 Subtraction - checking:

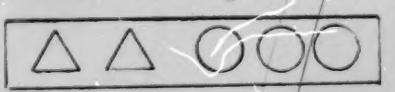
By addition - again stressing inverse relationships

$\begin{array}{r} 8 \\ -6 \\ \hline 2 \end{array}$ To check, add the bottom numbers. $\begin{array}{r} 6 \\ +2 \\ \hline 8 \end{array}$

(An alternative form: $\begin{array}{r} 8 \\ 6 \\ +\ 2 \end{array}$)

B-19 Family of facts:

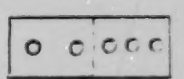
Help child recognize family of facts by showing how many number sentences can be written for a set of objects.



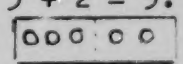
2 + 3 = 5
3 + 2 = 5

5 - 3 = 2
5 - 2 = 3

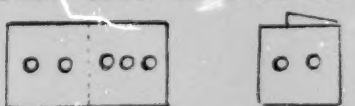
Domino cards are helpful at this point.



2 + 3 = 5
Turn card to show
3 + 2 = 5.



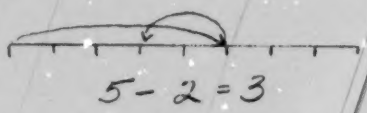
Fold card to show subtraction.



5 - 3 = 2
5 - 2 = 3

Introduce reinforcement and drill activities:

Possibilities:
Number wheels
Flash cards
Games and puzzles
Workbooks and worksheets
Numberline (to increase understanding of subtraction combinations)



Note: Introduce numeration strand objectives A3 through A6 before moving to next step.

B-33 Subtraction facts 10 through 19:

Stress using inverse relation between addition and subtraction to find differences.

$8 + 4 = 12$ $12 - 4 = 8$
$12 - 8 = 4$

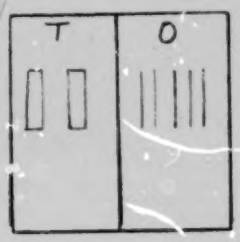
Note: The vertical form of subtraction $\begin{array}{r} 14 \\ -\ 8 \end{array}$ is difficult for many children at this stage. However, to follow the sequence of our textbooks, we do need to introduce it now.

Note: Manipulation of objects to form models of two-digit numbers (Numeration B-20) should precede the next step of subtraction.

B-39 Subtracting 1-digit from 2-digit numbers:

Present a story situation for subtracting 1-digit from 2-digit numbers. Example: There were 25 children playing and 4 went home. How many children stayed to play?

Set up counters on mat.



Remove 4 counters.

Count how many left.

How many tens?
How many ones?

Teacher may record on board or paper.

	tens	ones
	2	5
−		4
	2	1

Move from this object stage through picture and abstract stages as previously explained.

B-40 Subtracting 2-digit from 2-digit (no regrouping):

Follow progression through concrete, picture and abstract stages as outlined previously.

Attention should be given to this at concrete stage as preparation for regrouping.

$$\begin{array}{r} 34 \\ -\ 13 \\ \hline \end{array}$$

Remove 13 objects:
1 ten 3 ones
and record.

Note: In preparation for regrouping, various manipulatives may be used at the concrete stage. Popsicle sticks or Unifix cubes seem easiest to understand at beginning stages. Move from separate objects to metric/Dienes blocks or bean sticks, then to money. Poker chips (where 1 color of chip = 1, another color of chip = 10, and another color of chip = 100) could be used at a later stage.

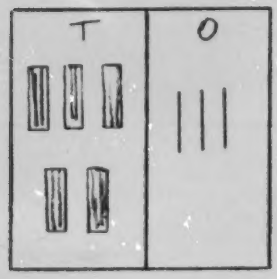
C-47 Introduction to renaming in subtraction:

Example: Give each child 5 tens and 3 ones. Children should place on their mats in appropriate places.

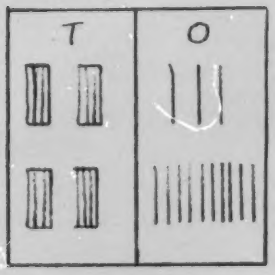
Children write numerals to indicate how many tens and ones. Ask children to unbundle one ten to get more ones. Place ones together. Record.

Note: This means that our work in numeration must not prevent the placement of ten or more in a given column.

Step 1



Step 2



"We trade 1 ten for 10 ones."

5	tens	3	ones
4	tens	13	ones

C-47 Subtract 1-digit from 2-digit (with renaming):

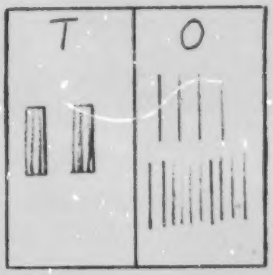
Present by telling a story: There were 34 children on the playground. Nine of them went inside. How many children are on the playground?

Step 1:

T	O
3	4
-	9

Child will use mat to show 34 in tens and ones. Ask what must be done before we can show that 9 of the children went inside. (Trade 1 ten for 10 ones.)

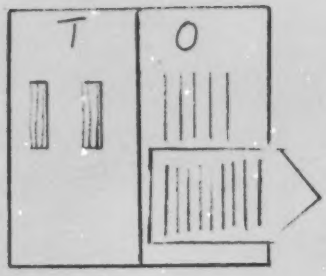
Step 2



T	O
2 3	14
-	9

Have them unbundle 1 bundle of 10 and put the single sticks in the ones place. Ask "How many tens and ones are there?" (2 tens and 14 ones) Teacher illustrates how to record action of regrouping.

Step 3

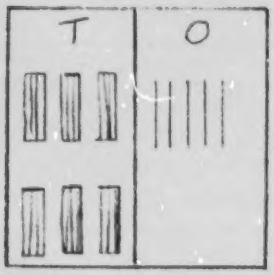


T	O
2 3	14
-	9
2	5

Ask students to remove 9 sticks from mat and count how many sticks remain. Count ones and record. Count tens and record. Read answer as 25.

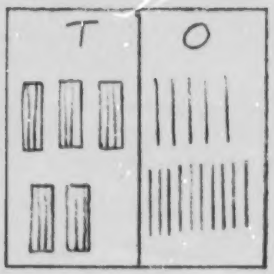
C-52
C-53 Continue through the same teaching sequence with any two 2-digit numbers, no zeros in minuend.

Step 1



65
-28

Step 2



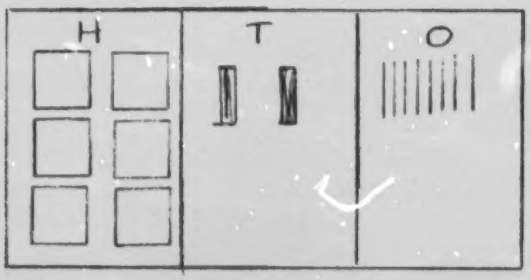
5
~~6~~ 5
-28

Step 3
Subtract and record.

Note: Introduce numeration strand objectives C-42 through 54 before moving to next step.

C-59 Subtract 3-digit numbers, no zeros in minuend, regrouping in tens only.

Step 1

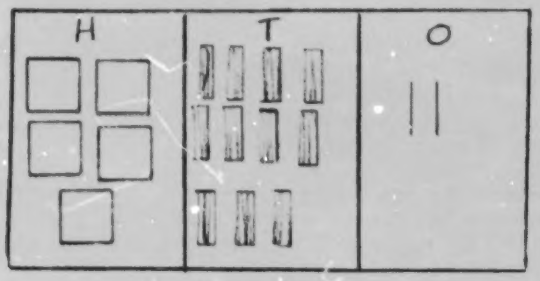


628
-146

Give students related examples, such as:

628
- 46

Steps 2 and 3



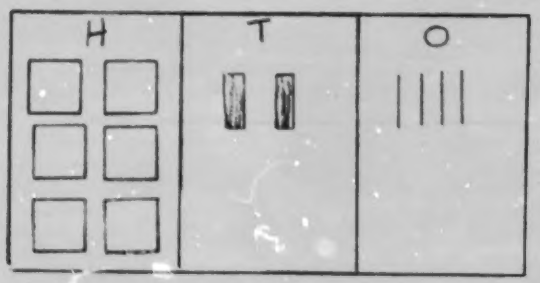
"Trade 1 one-hundred for 10 tens."

$$\begin{array}{r} {}^{5}\not{6}28 \\ -146 \\ \hline 2 \end{array}$$

Step 4
Remove tens and hundreds and record.

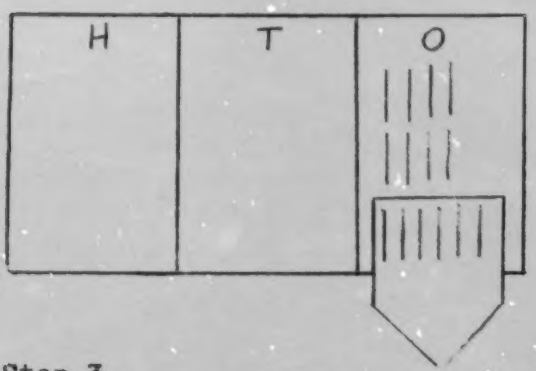
C-60 Subtract 3-digit numbers, no zeros in minuend, renaming in tens and ones:

Step 1



$$\begin{array}{r} 624 \\ -146 \\ \hline \end{array}$$

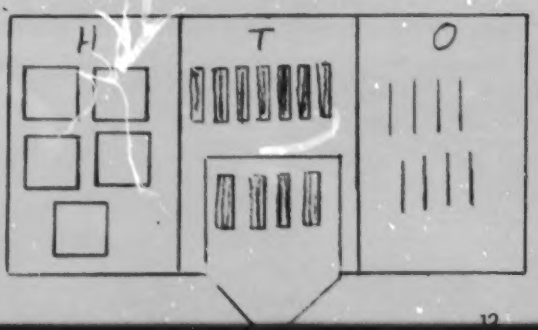
Step 2



"Trade 1 ten for 10 ones."

$$\begin{array}{r} 6{}^{1}\not{2}4 \\ -146 \\ \hline 8 \end{array}$$

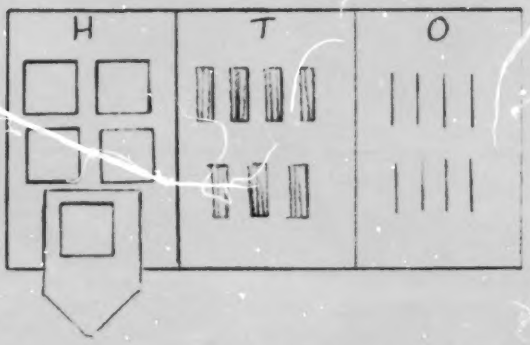
Step 3



"Trade 1 one-hundred for 10 tens."

$$\begin{array}{r} {}^{5}\not{6}{}^{11}\not{2}4 \\ -146 \\ \hline 78 \end{array}$$

Step 4



$$\begin{array}{r} {}^{5}\not{6}\,{}^{11}\not{2}\,{}^{1}4 \\ -\ 1\ 4\ 6 \\ \hline 4\ 7\ 8 \end{array}$$

Ask child to remove objects beginning in ones place. Count objects remaining and record in ones place. Do same for tens and hundreds. Finally, count all objects remaining on mat and compare with written answer.

C-61 Subtract any 3-digit numbers with zero in ones place only of minuend:

Examples: $\begin{array}{r} 560 \\ -225 \\ \hline \end{array}$ $\begin{array}{r} 510 \\ -225 \\ \hline \end{array}$

This objective can be modeled in the same manner as C-59 or C-60.

C-62 Subtract any 3-digit numbers with zero in tens place only of minuend:

Example: $\begin{array}{r} 508 \\ -126 \\ \hline \end{array}$

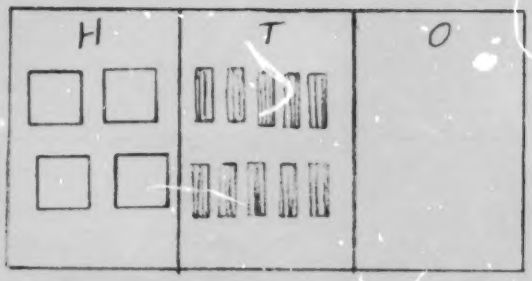
C-63 Subtract any 3-digit numbers with zero in ones and tens places of minuend:

Step 1

$$\begin{array}{r} 500 \\ -248 \\ \hline \end{array}$$

Child begins with model of 500. Ask child to remove 248 objects. Allow experimentation and discuss how this could be done.

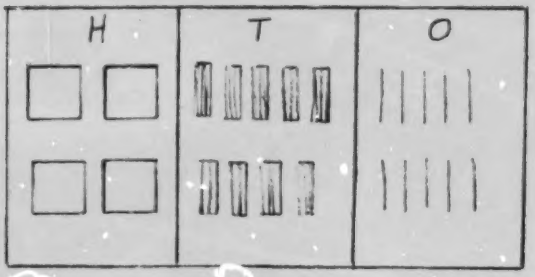
Step 2



$$\begin{array}{r} {}^{4}\not{5}00 \\ -248 \\ \hline \end{array}$$

Must begin by trading a hundred for 10 tens and recording.

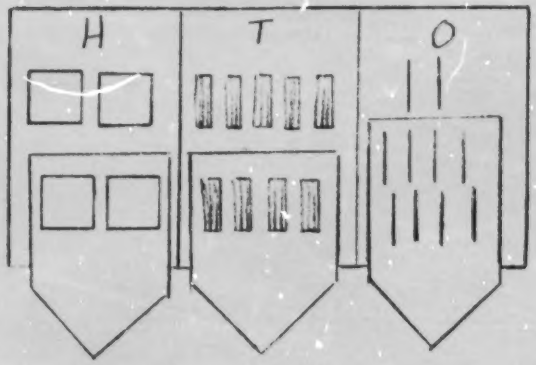
Step 3



$$\begin{array}{r} {}^{4}\not{5}\,{}^{9}\not{0}0 \\ -248 \\ \hline \end{array}$$

Trade a ten for 10 ones. Show how to record this trading.

Step 4



$$\begin{array}{r} {}^{4}\not{5}\,{}^{9}\not{0}0 \\ -248 \\ \hline 252 \end{array}$$

Begin removing sticks in ones place and record. Subtract in tens place and record. Subtract in hundreds place and record

Give students related examples, such as:

$$\begin{array}{rr} 500 & 500 \\ -128 & -\ 28 \end{array}$$

Alternate thinking pattern:
50 tens and no ones is same as 49 tens and 10 ones.

C-64 Subtract any 3 or 4-digit numbers:

Examples:

5065	4000	7305	4300
-2397	-2483	- 248	-1846

E-90 Subtract any two numbers < 1,000,000:

Examples:

408,369	870,003
- 246,983	- 54,691

MULTIPLICATION

Multiplication of whole numbers is approached through the concept of union of sets which are equal in number. If children can discover the need for repeated addition (multiplication) in real life situations, they will be motivated to learn the skill. If we continue to help them relate the skill to real life applications as they proceed from simple to more complex problems, they will be able to use the skill appropriately. This type of approach requires use of manipulatives and estimation.

The following guide presents a series of <u>suggested</u> sequential steps and strategies for teaching multiplication. They are based on the manipulative approach, giving the student opportunities to work with models. These models help the student with the discovery and understanding of the multiplication concepts and should be used in the beginning stages when numbers are small. The teacher should guide the student through the activities by asking appropriate questions. Throughout this guide sample questions and suggestions are listed in the column on the right of the pictures of the models. When they correspond, Des Moines Mathematics Performance Objective numbers are listed to the left.

During the several year period in which multiplication concepts are usually taught, the students are in the stage of concrete operations as defined by Piaget. (Children can reason logically if they have concrete manipulatives with which to test their thinking.) While the concepts are gradually being mastered, the children mature, and mathematics lessons progress from the concrete stage (real life situations and familiar objects) through the representational stage (pictures used in place of concrete objects) to the abstract (symbolism) stage in which drill is appropriate.

Before moving to the representational and abstract stages, various types of manipulatives (concrete objects) may be used; these manipulatives range from discrete objects (such as ice cream sticks or Unifix cubes) through items such as bean sticks and metric/Dienes blocks (non-discrete objects with physical quantity maintained) to money and poker chips (physical quantity not maintained -- higher value chip or bill has same physical size as lower value chip or bill). Teachers should choose manipulatives appropriate to the maturity of their students' thinking, neither boring students with too much emphasis on beginning stage manipulatives nor moving too fast to the abstract operation. For most students working with multiplication, metric/Dienes blocks and dot arrays are helpful manipulatives to use as each new step is begun.

Beginning level

Develop need from real life situations:

Sample questions:
Can you think of things that come in groups of two? (shoes, mittens, socks, eyes, ears, bicycle wheels)

Can we count how many there are in this group? Could we count by twos?

Similar questions for other sets -- tricycle wheels, animal legs, fingers, etc.

Note: A mat (construction paper, felt board, placemat, etc.) is used to distinguish and separate the objects used in the problem situation from other extraneous objects. This device helps avoid confusion in the student's mind. Students form sets of objects on their mats similar to those pictured in this guide.

A-3
A-6

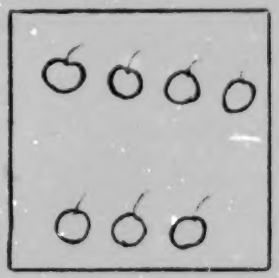
Recognize multiplication situation (using objects which aren't necessarily grouped in pairs, threes or any certain way):

This is 3 groups (or sets) of 2. 6 altogether.

3 times 2 is 6.

A-8 Relate multiplication to addition:

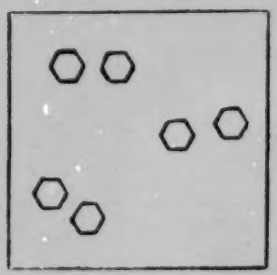
In both operations sets are joined. Multiplication is used only if sets are equal in number.



This shows 4 + 3.

This shows 3 + 3 + 3 + 3 and also 4 sets of 3 or 4 times 3.

Symbols - a way to write (record):



3 X 2 = 6
equation form

$$\begin{array}{r} 2 \\ \times 3 \\ \hline 6 \end{array}$$

computational form

Either of these forms can be used to record 3 times 2, meaning 3 sets of 2 each.

The answer is called the product. The two numbers being multiplied are called factors.

B-26 Simple story problems:

5 children. 3 cookies each. How many cookies altogether?

3 + 3 + 3 + 3 + 3 = ____

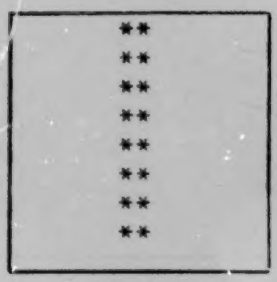
5 X 3 = _____

Given story problem (orally with objects or pictures), child writes appropriate equations.

Skip counting:

Count orally:

Help them start: "two, four, . . ."



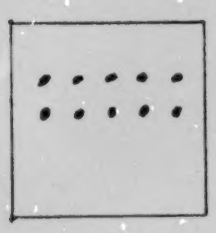
Count orally:

"three, six, . . ."

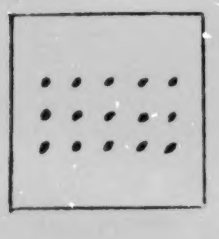
A-1
A-2
A-3
A-4
A-5
A-6
A-7
B-12a
B-24

Use of rectangular arrays to show multiplication:

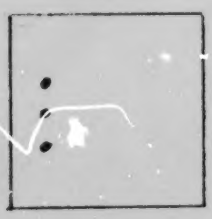
This shows a set of five objects in a row.



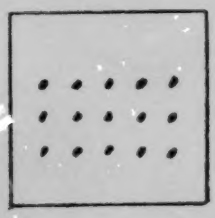
This shows two sets of five objects in two rows. 2 by 5 or 2 X 5 is 10. Rows will go from left to right, or right to left.



Sets of counters for multiplication problems may (and should) be arranged in rectangular arrays. This is a 3 by 5 array: 3 X 5 = 15.

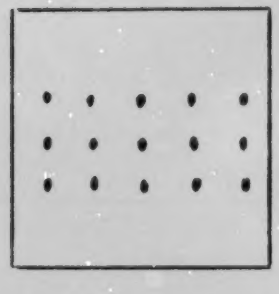


An array also has columns (from top to bottom of the group). A column has a counter from each row/set.



How may columns does this 3 by 5 array have? (5)
How many rows? (3)
How many sets? (3)

This 3 X 1 array has how many sets? (3)

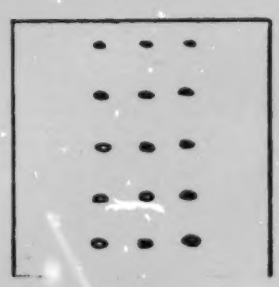


The equation (number sentence) for this array is
3 X 5 = 15.

Computational form is

$$\begin{array}{r} 5 \\ \times\ 3 \\ \hline 15 \end{array}$$

and is read "three times five."



This array shows 5 X 3 = 15.

Computational form is

$$\begin{array}{r} 3 \\ \times\ 5 \\ \hline 15 \end{array}$$

and is read "five times three."

Note: The committee urges teachers to present arrays in a consistent manner. Perhaps children cannot be expected to arrange arrays in a consistent manner in which each row is a set and the number in a column represents the number of sets, but they should not be confused by the teacher presenting conflicting models.

A-7 Interpretation of multiplication equation:

Give student a story problem and/or set model. Ask him/her to write equation.

2	X	6	=	12
↑		↑		↑
number of sets		number in each set		number in all the sets
↑		↑		↑
multiplier or factor		multiplicand or factor		product

In a multiplication equation the first factor tells the number of sets. The second factor tells the number in each set.

(A high level of mastery with regard to writing factors in correct order is not important.)

A-7 Interpretation of computational form for multiplication:

Give student a story problem and/or set model. Ask her/him to write computational form.

```
  6  ← number in each set
X 2  ← number of sets
 12  ← number in all the sets
```

In the computational form for multiplication, this factor (top) tells the number in each set. It is called the multiplicand. This number (second) tells the number of sets. It is called the multiplier. (It tells how many times the other number could be added.)

```
  6      means      6      not      2
X 2                +6               2
                                    2
                                    2
                                    2
                                   +2
```

> Note: This interpretation of vertical form is important in building rationale for multiplication by 2-digit numbers. Teachers should present it consistently, but they may accept the commutative form from students.

Zeros in multiplication (Empty set):

Can you show a row with nothing in it?
How about two rows?
How many objects in both rows?
Let's record. 2 X 0 = ___

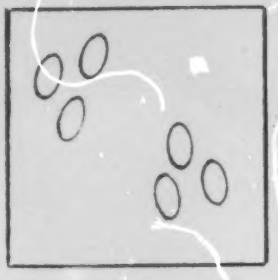
Show me no (zero) rows of 4.
0 X 4 = ____

What happens when zero is multiplied times any number?

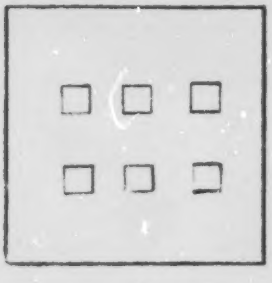
B-17 Mastery of multiplication facts:
B-18
B-19
B-20
B-21
B-22
B-25
C-29
C-30
C-31
C-33

> Note: Strategies for recalling basic facts vary from the immature counting by ones to more sophisticated techniques. It is important that children memorize facts. They learn simple facts easily, but they must be taught to use appropriate thinking strategies for solving and recalling the more difficult combinations. Each fact belongs to a group of facts for which one strategy is most appropriate. Each strategy must be introduced by a concrete model to which the student can refer mentally.

B-24 Models for multiplication:
C-27

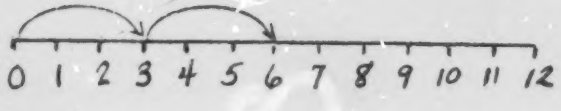


Set Model: 2 X 3 = 6



Array Model: 2 X 3 = 6

(We will use array to mean rectangular array. We also realize this is a unique type of set model.)

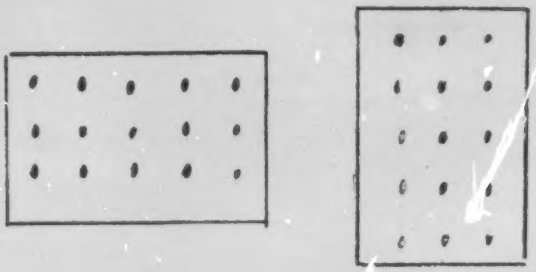


Numberline Model:

2 X 3 = 6

Strategies (using the recommended models):

Commutative Principle
(a X b = b X a)



3 X 5 = 5 X 3

Rotate dot array cards. Show that when two numbers are multiplied, the order of the numbers doesn't affect the product.

Show students that if they know one fact, they also know its partner. This concept cuts the number of facts which they need to learn.

Repeated Addition
or
Skip Counting

2,4,6,8,. . .

Can be modeled with number-line, set model or array

Use for teaching 2's, 5's, and possibly 3's and 4's.

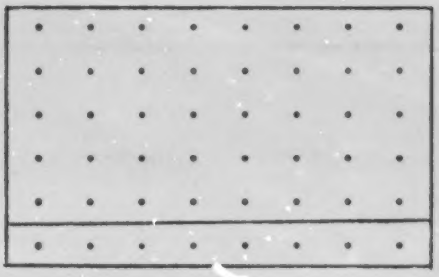
Note: The following three strategies are variations which use the distributive principle which relates multiplication and addition. These examples show how it works:

3 X (10 + 2) = (3 X 10) + (3 X 2)
a X (b + c) = (a X b) + (a X c)

(2 + 5) X 4 = (2 X 4) + (5 X 4)
(a + b) X c = (a X b) + (b X c)

These three strategies are taught best with array models. Encourage students to separate an array picture for an unknown fact into two facts which they do know. Don't expect students to write out formal equations to explain the logic.

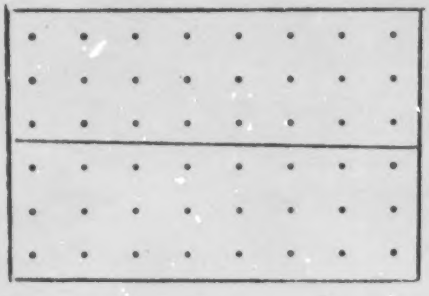
One-more-set



6 X 8 ⟶ (5 X 8) and one more set

40 + 8

Twice-as-much



3 X 8 = 24, so
6 X 8 is twice as much, or
24 + 24 = 48

This strategy is useful for teaching facts of 4, 6 and 8.

Using partial products

6 X 5 = 30 and 6 X 3 = 18
so
6 X 8 = 30 + 18 = 48

Note: Usefulness of these three distributive law techniques is limited to those times when addition or doubling does not require difficult renaming. Example: 3 X 7 = 21, so 6 X 7 = 42 is easy but, 4 X 7 = 28, so 8 X 7 = 56 is hard because the student must regroup "carry" in his head.

Patterns

Students examine a hundreds chart for patterns.

1	(2)	3	(4)	5	(6)	7	(8)	9	(10)
11	(12)	13	(14)	15	(16)	17	(18)	19	(20)

for 2's

1	2	3	4	(5)	6	7	8	9	(10)
11	12	13	14	(15)	16	17	18	19	(20)

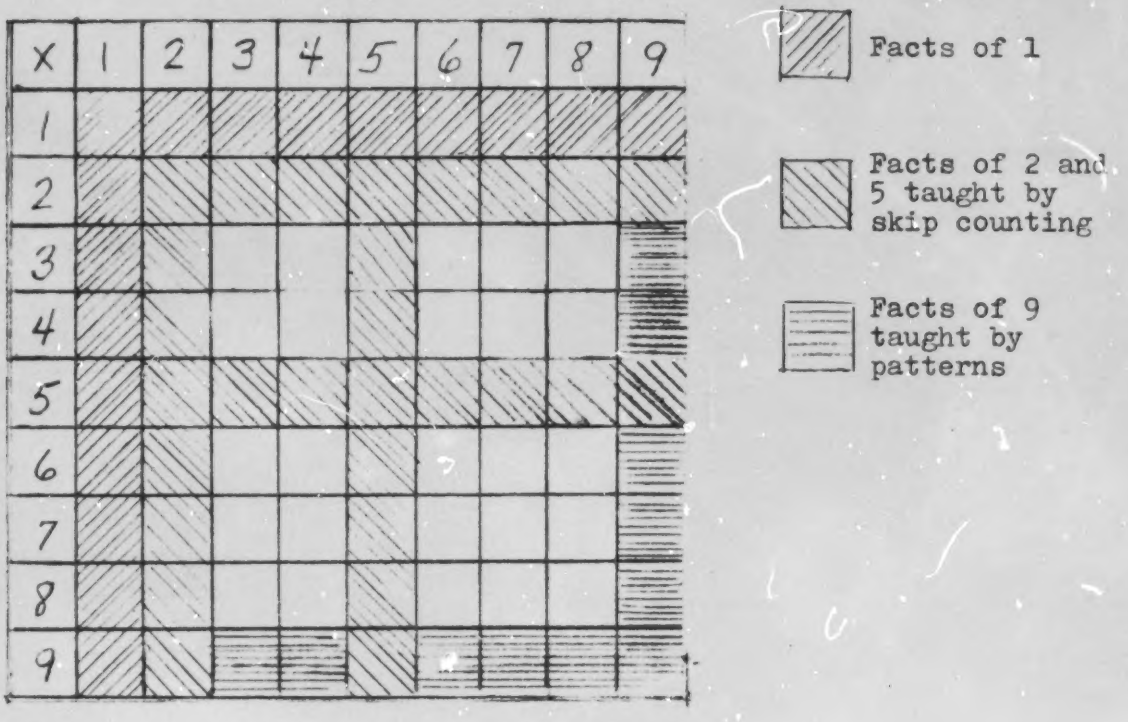
for 5's

1	2	3	4	5	6	7	(8)	(9)	10
							(18)		
						(27)			
					(36)				
				(45)					

for 9's (Sum of tens and ones digit = 9)

This strategy is especially effective to teach 9's.

Note: Motivate students to master facts by working on a small group of facts with an appropriate strategy. Let students mark off on their personal multiplication fact charts those facts which have been mastered. After the commutative property has eliminated nearly half of the table, the remaining task does not look so threatening.



Facts of 3 and 4 handled by repeated addition (or 3's by one-more-set and 4's by twice-as-much)

Square facts (6 X 6, 7 X 7, 8 X 8) are easy for some students.

6 X 7 can be done easily with twice-as-much strategy. (3 X 7 = 21, so 6 X 7 = 2 X 21, or 42)

6 X 8 can be done easily with one-more-set strategy. (5 X 8) + 8

Commutative facts eliminate all but one remaining pair of facts: (8 X 7); (7 X 8). Rote might be used.

C-53 Associative principle:

$(3 \times 2) \times 5 = 3 \times (2 \times 5)$
$6 \times 5 = 3 \times 10$
$30 = 30$

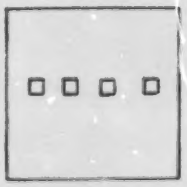
The way in which factors are grouped does not affect the product.

Review skip counting, including 10's and 100's:

> **Note:** The following stage in multiplication, involving multiplies of tens and hundreds, is crucial. It should be taught for a high level of mastery and reviewed periodically.
>
> We will show a "pattern" approach since modeling with manipulatives becomes burdensome with this concept. The commutative ($10 \times 2 = 2 \times 10$), associative ($[10 \times 2] \times 41 = 10 \times [2 \times 41]$) and distributive properties ($21 \times 34 = [20 + 1] \times 34$) are the bases for our procedure.
>
> Understanding multiplication by tens, hundreds, etc., will eliminate the meaningless row of zeros sometimes taught. e.g.
>
> $$\begin{array}{r} 42 \\ \times\ 30 \\ \hline 00 \\ 1260 \\ \hline 1260 \end{array}$$

C-41 Multiplication involving multiples
C-42 of tens and hundreds:
C-43
C-44
C-45
C-46

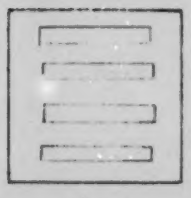
$4 \times 1 = 4$



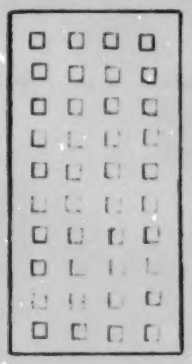
$1 \times 4 = 4$

Review this with the students.

Products of 10, 100, and 1-digit numbers:



4 X 10 =
4 tens, or 40



10 X 4 = 40

How are these arrays (4 X 10 and 10 X 4) like the other arrays? How are they different? How are their products different?

Product of multiples of 10, 100 and 1-digit numbers:

4 X 2 = 8	2 X 4 = 8
4 X 20 = 8 tens, or 80	20 X 4 = 80
4 X 200 = 8 hundreds, or 800	200 X 4 = 800

How are these examples alike? How are they different?

Product of two multiples of 10:

40 X 20 = 800	20 X 40 = 800

Product of 10, 100, 1000 and 2-digit numbers:

32 X 10 = 320

100 X 32 = 3200

32 X 1000 = 32000

Continue same line of questioning. Students should also use corresponding vertical forms.

Product of 10, 100, 1000 and 3-digit numbers:

462 X 10 = 4620

100 X 462 = 46200

462 X 1000 = 462000

Also: 10 X 462 462 X 100 etc.

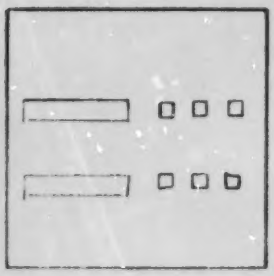
Product of multiples of 10 or 100 and 2 or 3-digit numbers:

20 X 41 = 820

321 X 400 = 128400

C-55 C-58 Multiplication of 2-digit by 1-digit:

Use metric/Dienes blocks.

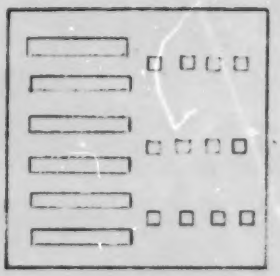


2 X 13

13
X 2
6
20
26

Graph paper could also be used. (See page 18.)

Students could multiply tens, then ones, but gradually lead them to work with the ones first.

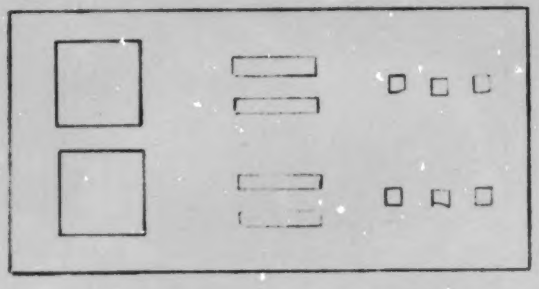


3 X 24

24
X 3
12
60
72

It is important to show both partial products at this stage.

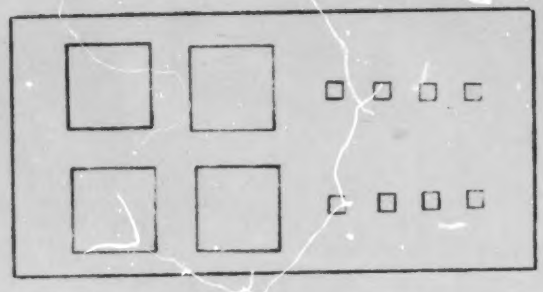
C-57
C-59 Multiplication of 3-digit by 1-digit:



2 X 123

$$\begin{array}{r} 123 \\ \times\ 2 \\ \hline 6 \\ 40 \\ 200 \\ \hline 246 \end{array}$$

Intermediate steps prior to the final algorithm.

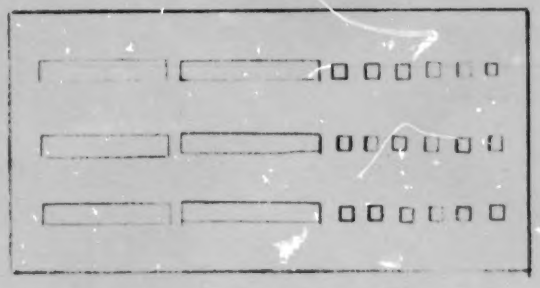


2 X 204

$$\begin{array}{r} 204 \\ \times\ 2 \\ \hline 8 \\ 400 \\ \hline 408 \end{array}$$

C-58 Regrouping:

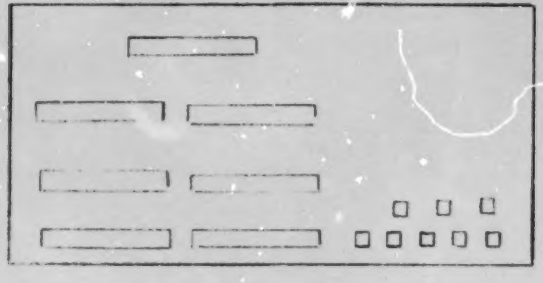
Step 1



3 X 26

$$\begin{array}{r} 26 \\ \times\ 3 \end{array}$$

Step 2



Trade 10 ones for 1 ten and record. (Note alternate choice of place to record.)

$$\begin{array}{r} {\scriptstyle 1} \\ 26 \\ \times\ 3 \\ \hline 8 \end{array} \quad \text{or} \quad \begin{array}{r} 26 \\ \times\ 3 \\ \hline {\scriptstyle 1} \\ 8 \end{array}$$

Step 3

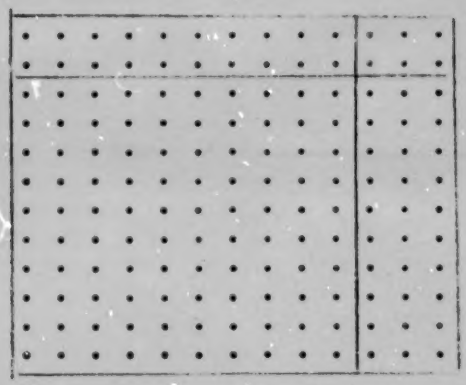
Next, combine the tens and record:

$$\begin{array}{r} {\scriptstyle 1} \\ 26 \\ \times\ 3 \\ \hline 78 \end{array} \quad \text{or} \quad \begin{array}{r} 26 \\ \times\ 3 \\ \hline {\scriptstyle 1} \\ 78 \end{array}$$

C-60 Multiplication of 3-digit by
C-61 1-digit numbers (with regrouping):

Use similar procedure as outlined for 2-digit numbers.

D-75 Multiplication of 2-digit by 2-digit:



12 X 13

$$\begin{array}{r} 13 \\ \times\ 12 \\ \hline 6 \\ 20 \\ 30 \\ 100 \\ \hline 156 \end{array}$$

2 X 3
2 X 10
10 X 3
10 X 10

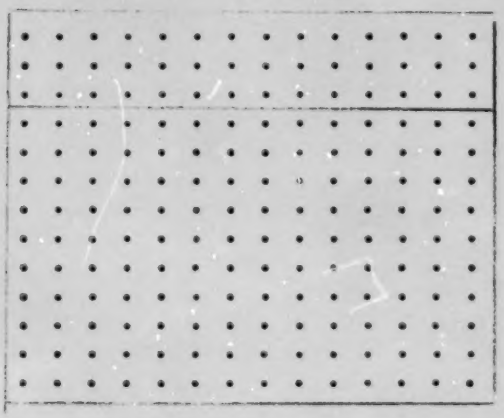
Large dot arrays or graph paper can be used for modeling. They are very flexible, especially for 2-digit factors. Use a large sheet and cover the unwanted portion.

(When using graph paper some students may need to see a dot in each square. Listen to the students' discussion to be certain that they are thinking about the squares and not the points where the lines intersect.)

This model uses same thinking as the distributive law partial product model which was used to develop multiplication facts. (Multiplication, page 10)

This is an introductory stage used to develop understanding. Use it, but don't teach for high level of mastery.

D-77 Simplifying the algorithm:



13 X 14

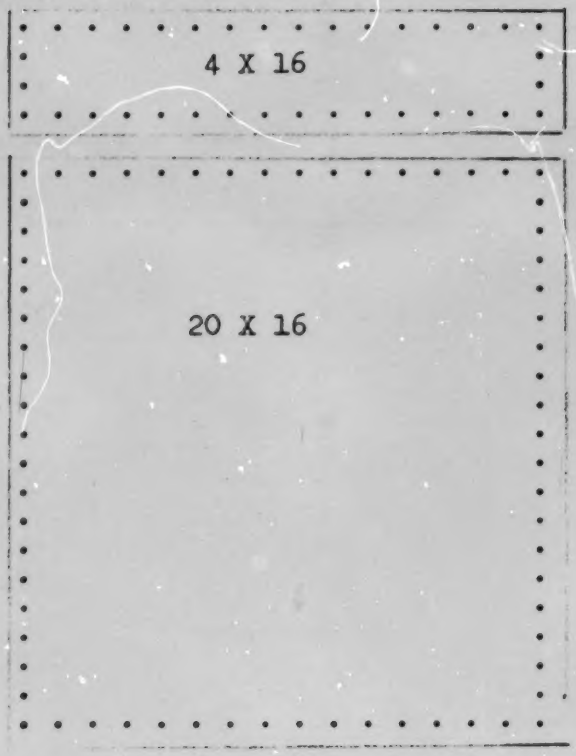
$$\begin{array}{r} 14 \\ \times\ 13 \\ \hline 42 \\ 140 \\ \hline 182 \end{array}$$

3 X 14
10 X 14

This dot array model has been changed to show just two partial products. Therefore, we can condense the recording.

In this problem there are 3 sets of 14 and 10 sets of 14. We multiply 14 by the number in the ones place, and then we multiply 14 by the number of tens.

Students may need to review procedure for multiplying 2 and 3-digit numbers by 1-digit numbers, especially with regrouping.



This time we use 20 as the multiplier for the second partial product.

Students may need to review multiplication involving multiples of tens and hundreds. (See Multiplication, pages 13-15.)

24 X 16

$$\begin{array}{r} 16 \\ \times\ 24 \\ \hline 64 \\ 320 \\ \hline 384 \end{array}$$

$$\begin{array}{r} 1 \\ 2 \\ 16 \\ \times\ 24 \\ \hline 64 \\ 320 \\ \hline 384 \end{array}$$

Special attention will need to be given to numbers which are "carried", especially if "carrying" is involved in both partial products.

$$\begin{array}{r} 16 \\ \times\ 24 \\ \hline 64 \\ 320 \\ \hline 384 \end{array}$$

It may be easier to use alternate suggestion from page 17.

52

The mature algorithm:

48 X 76

```
  76
X 48
 288
144     omit zero
1728
```

This represents the final abstract level after progression from the concrete model through developmental work. If the student cannot master this, it will be necessary to do a diagnostic analysis. (Does he understand the concepts? At what level does his understanding break down? Does he know the basic facts? Does he get confused and forget what he is doing? Is place value a problem? etc.)

If a student has trouble with the mechanics of this step, it is sometimes easier to multiply 3-digit numbers by 1-digit numbers before insisting on mastery.

Checking:

To review rationale, make dot arrays for a couple of problems, then rotate the dot arrays. (Caution: choose problems with factors not larger than 25 so the model isn't too cumbersome.)

```
  57          42
X 42        X 57
 114         294
228         210
2394        2394
```

Use the commutative principle to check.

D-78 Multiply 3-digit number
D-79 by 2-digit number:

D-80 Multiply 3-digit number
by 3-digit number:

DIVISION

Division of whole numbers can be approached in several ways, one of which is separating a set into subsets which are equal in number. This paper addresses itself to that approach. Comparisons of two sets, finding fractional parts, division of remainders, etc. will be developed in another paper.

Development of division concepts is built on the students' understandings of numeration, multiplication, subtraction and their interrelationships. Students should discover the need for division in real life situations. Early concepts should grow out of the students' work with numeration and other algorithms and progress through stages to the final abstract algorithm.

The following guide presents a series of <u>suggested</u> sequential steps and strategies for teaching division. They are based on the manipulative approach, giving the student opportunities to work with models. These models help the student with the discovery and understanding of the division concepts and should be used in the beginning stages when the numbers are small. The teacher should guide the student through the activities by asking appropriate questions. Throughout this guide sample questions and suggestions are listed in the column on the right of the pictures of the models. When they correspond, Des Moines Mathematics Performance Objective numbers are listed to the left.

During the several year period in which division concepts are usually taught, the students are in the stage of concrete operations as defined by Piaget. (Children can reason logically if they have concrete manipulatives with which to test their thinking.) While the concepts are gradually being mastered, the children mature, and mathematics lessons progress from the concrete stage (real life situations and familiar objects) through the representational stage (pictures used in place of concrete objects) to the abstract symbolism stage in which drill is appropriate.

Before moving to the representational and abstract stages, various types of manipulatives (concrete objects) may be used; these manipulatives range from discrete objects (such as ice cream sticks or Unifix cubes) through items such as bean sticks and metric/Dienes blocks (non-discrete objects with physical quantity maintained) to money and poker chips (physical quantity not maintained -- higher value chip or bill has same physical size as lower value chip or bill). Teachers should choose manipulatives appropriate to the maturity of their students' thinking, neither boring students with too much emphasis on beginning stage manipulatives nor moving too fast to the abstract operations.

Division is the last of the four major operations to be presented; therefore, the manipulative experience may not need to be as extensive as for the operations which are presented earlier. For many students working with division, metric/Dienes blocks are a useful tool. For some, especially those who reject blocks as babyish, more abstract items such as money or poker chips give adequate support and provide motivation. A few students could begin with the representational type pictures of models and move more rapidly toward the abstract. Teachers need to be alert to depth of the students' understandings and judge carefully when to discontinue their use. This timing varies from student to student.

Beginning level

Develop need from real life situations:

Sample questions:
How many mittens in a pair?
How many pairs (or groups of two) in this pile of mittens (shoes, socks, earrings)

Similar questions for other sized sets - sticks of gum, toy car wheels, triangle points, animal legs, etc.

Here are some pencils. Count them. If we divide them among three people, how many pencils will each person get? two people? etc.

Note: Division requires two ways of thinking: how many groups (sets) of a certain size, and how many in each of a specified number of groups (sets). Real life situations present both ways of thinking. Models can be used to represent both ways of thinking. The abstract computational form or number sentence can be interpretated either way. We urge teachers to include both approaches in the development of division.

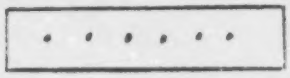
Note: A mat (construction paper, felt board, placemat, etc.) is used to distinguish and separate the objects used in the problem situation from other extraneous objects. This device helps avoid confusion in the student's mind. Students form sets of objects on their mats similar to those pictured in this guide.

A-5
A-10
A-11

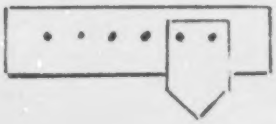
Recognize need for separation and arrange set of manipulatives into subsets: (Identify answer in terms of the story problem.)

Find the number of groups.

original model



step 1



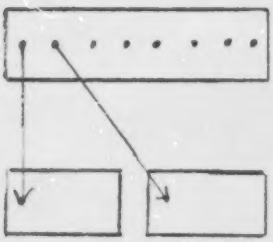
Here are 6 pieces of candy. How many sets of 2 candies each can you make?

(We made 3 sets of candies.)

Find number in each of a given number of groups.

original model

step 1



Here are 8 pieces of candy. Will you put them into 2 sets to see how many will be in each set? Please put the same number in each set.

(There are 4 candies in each set.)

Division is used to separate a set into subsets which are equal in number.

Symbols - a way to write (record):

6 pieces of candy. Each girl can have 2 pieces. How many girls can have candy?

This problem is similar to a multiplication problem, but one of the factors is missing.

Write ____ X 2 = 6.

Separate the set of candies to find the number of subsets.

We can also write 6 ÷ 2 = ____. Read this, "6 divided by 2 equals what number?"

8 pieces of candy. Two boys share the candy. How many pieces of candy for each boy?

This problem can be written 2 X ____ = 8 or 8 ÷ 2 = _____.

Did you notice that we separate the candy in both problems, but we found different kinds of answers? In the first one we found the number of girls (the number of sets of candy). In the second one we found the number of pieces of candy for each boy (the number of pieces in each subset).

Have students write equations for simple story problems.

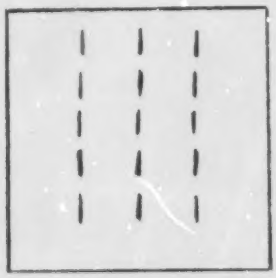
Note: Students should be able to see the two types of division models. If they have difficulty seeing the difference, use problems in which one factor is rather large and the solution is not obvious. Set models for _____ X 23 = 46 and for 23 X ____ = 46 are quite different.

A-1
A-2
A-3
A-4
A-6
A-7
B-12a

Rectangular arrays to show multiplication:

5 sets of pencils. 3 pencils in each set. How many pencils altogether?

Put pencils on mat (3 at a time) in a rectangular array.



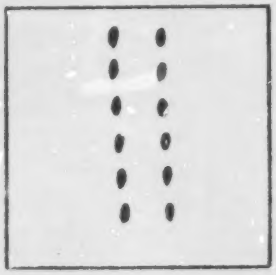
You have learned to show multiplication ideas with rectangular arrays.

This 5 by 3 array shows that 5 sets of 3 objects is 15. The equation is $5 \times 3 = 15$

A-1
A-3
A-4
A-5
A-10

Rectangular arrays to show division:

12 pieces of candy. Each girl can have 2 pieces. How many girls can have candy?



Arrays can also show division.

Make an array on your mat putting each set of candy in a row. Rows go across left to right.

How many rows (sets) are in the array?

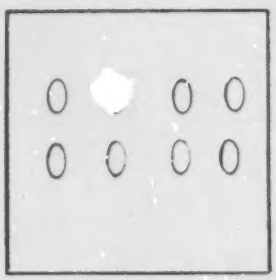
Teacher writes

and asks student to write answer in the box.

The multiplication sentence is $6 \times 2 = 12$.
This idea can also be written $12 \div 2 = 6$.

In this problem we were looking for the number of sets (rows) of candy.

8 pieces of candy. Two boys share the candy. How many pieces of candy for each boy?



Make an array on your mat with a row (set) of candy for each boy.

How many pieces of candy are in each row?

When you look at the array you also see columns which go up and down. There are the same number of columns as there are objects in each row.

How many objects in each set? How many columns in the array?

Teacher writes $2\overline{)\square}$

and asks student to write answer in the box.

The multiplication sentence is $2 \times \underline{4} = 8$. This idea can also be written $8 \div 2 = \underline{4}$.

In this problem we were looking for the number in each set.

B-15
B-26
C-35
D-86

Interpretation of multiplication equation:

$2 \times 7 = 14$

number of sets	number in each set	number in all the sets
multiplier or factor	multiplicand or factor	product

Review multiplication lessons for objective A-7, Multiplication page 6.

$__ \times 7 = 14$

Let's make up a story problem for this equation.

There should be 14 objects of some kind, and we will put 7 of them in each set. We will look for the number of sets.

$2 \times __ = 14$

Let's make up a story problem for this equation. There will be 14 objects of some kind, and we will separate them into 2 sets. We will find the number of objects in each set.

B-15
C-35
D-87

Interpretation of division equation:

$15 \div 5 = \frac{\quad}{\quad}$

dividend	divisor	quotient

The number being divided is called the dividend. It is divided by the divisor. The answer is called the quotient.
(A high level of mastery for this vocabulary is not necessary.)

$15 \div 5 = \underline{3}$ could mean

$5X ____ = 15$

or

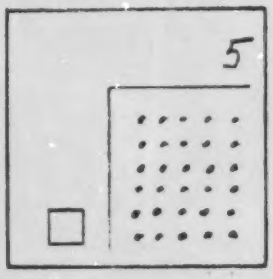
$____ X 5 = 15$

Let's make up a story problem for this equation.

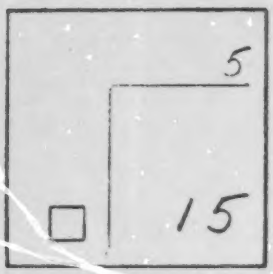
Which kind of problem will it be? Do we know how many sets to make, or do we know how many objects to put into each set?

$15 \div 5 = \underline{3}$ could be used for either type of problem.

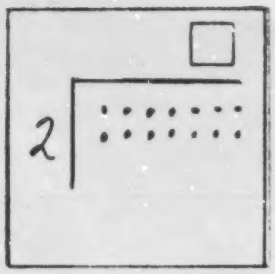
C-35 C-48 Developmental step leading to computational (column) form:



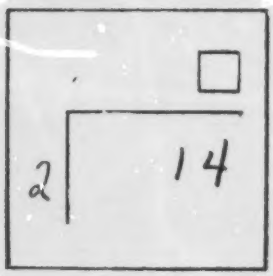
How many sets of 5 are in this array? (Student responds by placing numeral in box.)



Let's write the number of objects instead of showing them. How many sets (rows) of 5 would there be in the array? (Student responds by placing numeral in the box.)

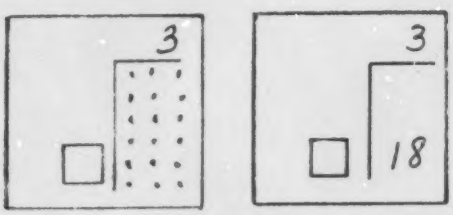


How many objects in each of two sets? (Student responds by placing numeral in the box.)

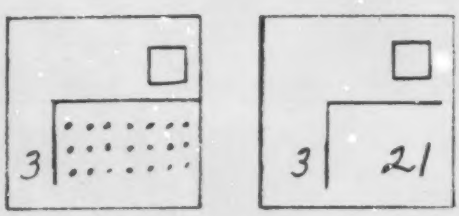


Again, let's write the number of objects instead of showing them. How many objects would be in each set (row)? (Student responds by placing numeral in the box.)

C-35 C-48 Computational (column) form for division:



How many sets of 3?



How many in each of 3 sets?

$$4 \overline{)20}\quad 5$$

This is standard computational (or column form) for both types of problems: (1) finding the number of sets or (2) finding the number in each set. It means the same as the number sentence $20 \div 4 = ____$.

Mathematicians do not write

$$5 \overline{)20}\quad 4$$

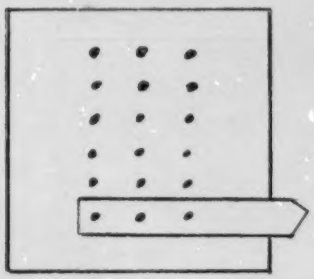
C-36 Easy basic facts:
C-37
C-38 Review the multiplication
C-39 families of facts and study the related division facts.

3 X 2 = 6	2 X 3 = 6
3 X __ = 6	2 X __ = 6
6 ÷ 3 = __	6 ÷ 2 = __

Help students achieve mastery. Keep record of which facts have been mastered and of the amount of time which is required to answer them (orally or written).

A-12 Division by 1-digit divisors - Repeated subtraction model (Finding the number of sets):

step 1



Ask student how many sets of 3 can be found in 18. Using manipulatives, separate sets one at a time.

Record:

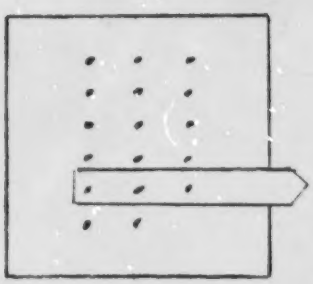
```
 18
- 3
 15
- 3
 12
- 3
  9
- 3
  6
- 3
  3
- 3
  0
```

There were 6 sets. We subtracted 3, 6 times.

Observe that the process of subtraction is the same whether or not the objects had been in a rectangular array.

Remainders:

step 1



How many sets of 3 are in 17?

How many objects are left (remain)?

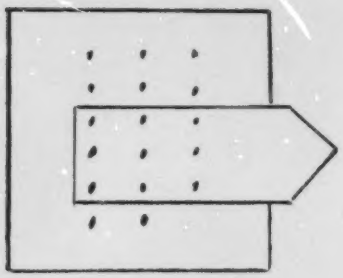
Record:

```
 17
- 3
 14
- 3
 11
- 3
  8
- 3
  5
- 3
  2
```

There are 5 sets of 3 in 17, and 2 objects remain.

A-12 Condensing the steps:

step 1



Several subtractions could be done at one time. How many sets of 3 shall we remove the first time?

$$\begin{array}{r} 17 \\ -\ 9 \\ \hline 8 \end{array} \quad 3 \times 3$$

$$\begin{array}{r} -\ 6 \\ \hline 2 \end{array} \quad \begin{array}{r} 2 \\ \hline 5 \end{array} \times 3$$

5 sets were subtracted, and there were 2 objects remaining.

Could we have subtracted 5 sets the first time?

$$\begin{array}{r} 17 \\ -15 \\ \hline 2 \end{array} \quad 5 \times 3$$

C-40 Using conventional notation:

Substitute

$$3\overline{)\begin{array}{r} 5 \\ 17 \\ -15 \\ \hline 2 \end{array}} \quad \text{for} \quad \begin{array}{r} 17 \\ -15 \\ \hline 2 \end{array} \quad 5 \times 3$$

This is another way to write division.

Since 5 means 5 ones, it should be written in ones place, above the 7.

Logical reasoning and estimation:

$$3\overline{)12}\ \ 4 \qquad 3\overline{)13}\ \ 4\text{r}1 \qquad 3\overline{)14}\ \ 4\text{r}2 \qquad 3\overline{)15}\ \ 5$$

$$5\overline{)35}\ \ 7 \qquad 5\overline{)36}\ \ 7\text{r}1 \qquad 5\overline{)37}\ \ 7\text{r}2 \qquad 5\overline{)38}\ \ 7\text{r}3$$

$$5\overline{)39}\ \ 7\text{r}4 \qquad 5\overline{)40}\ \ 8$$

Use multiplication facts as an aid in developing the division algorithm.

Give students problems in series such as these.

Remainders - Limits on size:

$$\begin{array}{r} 7\text{r}7 \\ 5\overline{)42} \\ \underline{35} \\ 7 \end{array}$$

How large is this remainder? Is it large enough for another set of 5? Usually, division examples ask for the largest possible quotient. Is 7 the greatest possible quotient?

Remainder - Identifying (labeling) in the story problem:

$$\begin{array}{r} 5 \\ 3\overline{)17} \\ \underline{15} \\ 2 \end{array}$$

Remainders can be interpreted in terms of real life situations. Mathematically, there is no whole number solution to the equation n X 3 = 17, but in real life situations there are several interpretations possible.

17 candies. Put 3 candies into each bag. How many bags? How many candies are left?

5 bags, 2 candies

17 candies to be packaged in 3 bags. How many candies in each bag? How many left?

5 candies, 2 candies

17 inches of ribbon. Cut into 3 equal lengths. How long is each piece?

5 2/3 inches

5 candies per bag. How many bags must be purchased to get 17 candies?

4 bags

The teacher should continue to relate the division algorithm to real life applications.

D-70 Two-digit quotients - Using subtractive method:

```
2 / 28
  -20      10 X 2
    8
  - 8      4 X 2
    0      14
```

Repeated subtraction is good for development of understanding, but it is time consuming and is not a process to be taught for a high level of skill. Lead students by questioning or suggestion to subtract 10 sets (or multiples of 10 sets) whenever possible. (Use manipulatives if there is confusion.)

```
2 / 49
  -20      10 X 2
   29
  -20      10 X 2
    9
  - 8      4 X 2
    1      24
```

Could this be simplified more?

```
2 / 49
  -40      20 X 2
    9
  - 8      4 X 2
    1      24
```

When students begin to think in terms of tens (or multiples of tens) of sets, they are actually looking for the missing factor. (What number times 2 is less than or equal to 49? How many times can 2 be subtracted from 49?)

Use the subtractive idea with divisors of 5 or less and quotients of 30 or less. (Problems with 2-digit divisors and quotients up to 5 or 6 could also be modeled easily, but to do so at this time would not be consistent with the sequence in most text-books.)

As soon as students can handle these ideas and when they are ready for larger divisors and quotients, switch emphasis to multiplication and missing factor concepts.

Note: Multiplication involving multiples of tens and hundreds (Multiplication-Division Objective Numbers C-41 - 46) should be taught before going on to the next step. (See Multiplication page 13.)

Finding the missing factor:

$4 \times 10 = 40 \rightarrow 40 \div 4 = 10$ or $4\overline{)40}^{10}$

$4 \times 20 = 80 \rightarrow 80 \div 4 = 20$ or $4\overline{)80}^{20}$

$40 \times 20 = 800 \rightarrow 800 \div 40 = 20$ or $40\overline{)800}^{20}$

Develop a high level of skill with this type of exercise.

D-70 Developmental step directed toward recording quotient in conventional way:

$$\begin{array}{r} 32\text{r}1 \\ 2 \\ 30 \\ 2\overline{)65} \\ -60 \\ \hline 5 \\ -\ 4 \\ \hline 1 \end{array}$$

or

$$\begin{array}{r} 30 \\ 2\overline{)65} \\ -60 \\ \hline 5 \end{array}$$

We can write the number of sets on top instead of on the side.

Guide students to write the ones in ones place and the tens in tens place.

leading to:

$$\begin{array}{r} 32\text{r}1 \\ 2\overline{)65} \\ -60 \\ \hline 5 \\ -\ 4 \\ \hline 1 \end{array}$$

leading to:

$$\begin{array}{r} 32\text{r}1 \\ 2\overline{)65} \\ -60 \\ \hline 5 \\ -\ 4 \\ \hline 1 \end{array}$$

C-56 Checking division - Use of inverse relationships:

$$\begin{array}{r} 4 \\ 3\overline{)14} \\ -12 \\ \hline 2 \end{array}$$

check:

$$\begin{array}{r} 3 \\ \times\ 4 \\ \hline 12 \\ +\ 2 \\ \hline 14 \end{array}$$

Illustrate both the division and the check with models.

$12 \div 3 = 4 \rightarrow 4 \times 3 = 12$

$14 - 2 = 12 \rightarrow 12 + 2 = 14$

Division and multiplication are inverse operations. This means that each operation undoes the other. Take a set apart by division. Put it back together with multiplication.

Subtraction and addition are also inverse operations. Take objects away with subtraction. Put them back with addition.

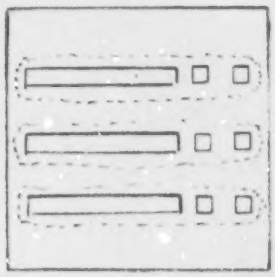
Use these ideas to check division work. Separate the set by division and subtraction. Then, to check, put it back together with multiplication and addition. The answer should be the same as the beginning number.

Note: Multiplication of 2 and 3-digit numbers by 1-digit numbers (Multiplication-Division Objective Numbers C-55 and C-57 should be taught before going to next step. (Refer to Multiplication pages 15-16.)

D-70 Two and three-digit quotients -
D-71 Partitioning model (Finding the number in a set):

Repeated subtraction becomes difficult to model as the quotients become larger. It is easier to model these steps with a partitioning model.

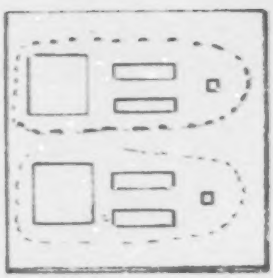
Using metric/Dienes blocks (or ten dollar and one dollar bills or poker chips of 2 different colors):



$$3 \overline{)\ 3 \text{ tens } 6 \text{ ones}}\ \ 1 \text{ ten } 2 \text{ ones}$$

Separate into 3 equal sized groups. How many in each group?

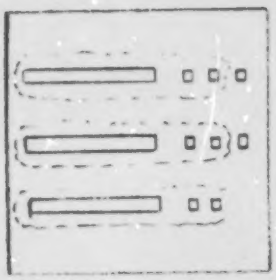
Guide the students to separate (divide) the tens first. At this stage it isn't necessary and to do so will seem awkward.



$$2 \overline{)\ 2 \text{ hundreds } 4 \text{ tens } 2 \text{ ones}}\ \ 1 \text{ hundred } 2 \text{ tens } 1 \text{ one}$$

Guide students to divide hundreds first, then tens, then ones.

Remainders



1 ten 2 ones r 2 ones

3 / 3 tens 8 ones

-3

8

-6

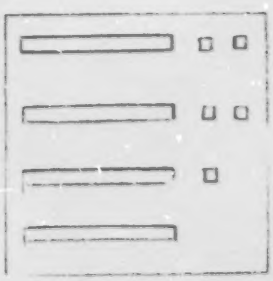
2

Because of the remainder, a more complete recording is necessary.

Note: Renaming ("carrying") in multiplication (Multiplication - Division Objective Numbers C-58 - C-61) must be taught before going on to the next step. See Multiplication page 17.

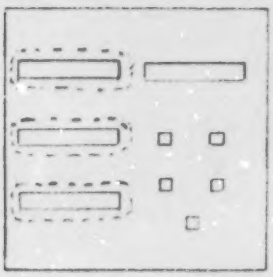
Renaming:

Step 1



3 / 4 tens 5 ones

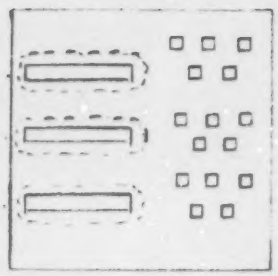
Step 2



Separate the tens, making 3 sets. 1 ten will be in each set.

$$3 \overline{)\ 4 \text{ tens } 5 \text{ ones}}$$
$$-3$$
$$1$$

Step 3

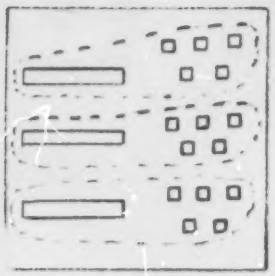


There aren't enough tens left to give one ten to each of the 3 sets, so:

Trade the remaining ten for 10 ones.
10 ones + 5 ones = 15 ones.

$$3 \overline{)\ \begin{matrix}1\\4 \text{ tens } 5 \text{ ones}\end{matrix}}$$
$$-3$$
$$1 \qquad 15$$

Step 4



Separate the ones, making 3 sets with 5 ones in each.

```
        1 ten  5 ones
3 /  4 tens 5 ones
   -3
   ---------------
    1     15
          15
          --
```

It may be necessary to use this same approach with a 3 digit dividend. More steps will be involved.

Abstract form:

```
     Tens | Ones
       1  |  2
3 /   3  |  6
    - 3  |
    -----|----
         |  6
       - |  6
    -----|----
         |
```

```
      H | T | O
      1 | 2 | 1
2 /   2 | 4 | 2
    - 2 |   |
    ----|---|---
        | 4 |
      - | 4 |
    ----|---|---
        |   | 2
      - |   | 2
    ----|---|---
        |   |
```

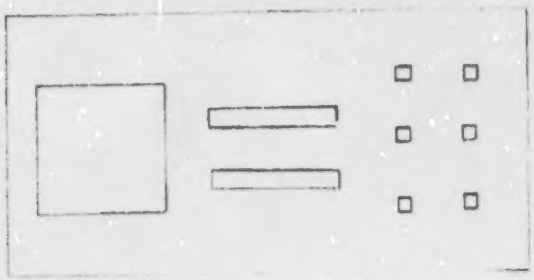
This grid form is a quicker way to write the problems which we just did.

```
      T | O
      1 | 2  r 2
3 /   3 | 8
    - 3 |
    ----|----
        | 8
      - | 6
    ----|----
        | 2
```

```
      T | O
      1 | 5
3 /   4 | 5
    - 3 |
    ----|----
        | 1 5
    - 1 | 5
    ----|----
        |
```

D-71 Renaming and/or estimating:

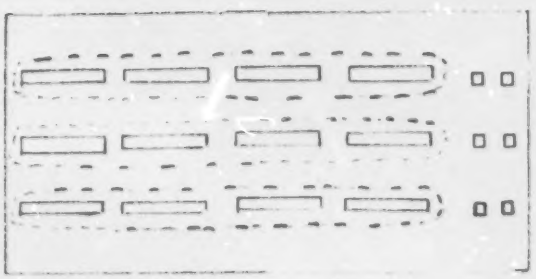
Step 1



There aren't enough hundreds to put one hundred in each of 3 sets, so:

	Hundreds	Tens	Ones
3 /	1	2	6

Steps 2 and 3

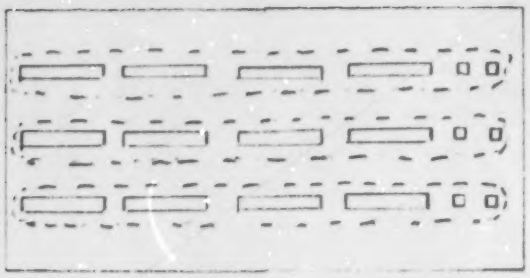


Trade 1 hundred for 10 tens.
10 tens + 2 tens = 12 tens.

Separate the tens; put 4 tens into each set.

	Tens	Ones
	4	
3 /	1 2	6
	1 2	

Step 4



Separate the ones into 3 sets.

	Tens	Ones
	4	2
3 /	12	6
	12	
		6
		6

Notice that estimation would help judge size of quotient. In this example, there weren't enough hundreds to share among 3 groups. 1 hundred for each group (or 100 X 3) would require 300. Therefore, an immediate judgment would be that the quotient would have only 2 places. (This thinking corresponds to the traditional "decide" step in the mature division algorithm.

Think 12 tens ÷ 3. The missing factor concept is helpful.

____ X 3 = 12
____ X 3 = 120
____ X 3 = 1200

This thinking corresponds to the traditional "divide" for trial quotient step of the mature division algorithm.

Relating the partitioning and subtractive interpretations of division:

Tens	Ones
4	2
3/ 12	6
12	
	6
	6

```
      42
       2
      40
3 / 126
   -120      40 X 3
      6
     -6       2 X 3
```

These two forms of division are equivalent:
(1) the "grid" form which was developed by partitioning Dienes blocks (or money or chips) and
(2) the form which used repeated subtraction.

D-72 Zeros in division:

Models may be used for these problems similar to those used in previous examples.

	Hundreds	Tens	Ones
	2	0	
4 /	8	0	4
	8		
		0	

There are enough hundreds to separate into 4 groups. There are no tens, so therefore, no tens will be separated. Proceed to separating ones.

	Hundreds	Tens	Ones
	2	0	
4 /	8	2	4
	8		
		2	4

In this problem again, there are enough hundreds to separate into 4 groups. This time, however, there are tens, but not enough to separate tens into 4 groups, so the 2 tens are renamed. 2 tens = 20 ones. 20 ones + 4 ones = 24 ones. Proceed to separating ones.

	Hundreds	Tens	Ones
	2		
4 /	9	0	4
	8		
	1	0	

Separating the hundreds into 4 groups does not use all the hundreds. Rename the remaining hundred as 10 tens. There will now

be tens to separate into 4 groups even though the dividend has zero in the tens place.

D-84 $0 \overline{)\ 6}$ meaningless

Division by zero is not possible because there is no factor times zero which equals 6 - or anything but zero.

$\underline{????} \times 0 = 6$

D-63 D-64 D-65 D-66 D-68 **More difficult basic facts:**

Review the multiplication families of facts and study the related division facts.

$6 \times 8 = 48$ $\quad$ $8 \times 6 = 48$

$6 \times _ = 48$ $\quad$ $8 \times _ = 48$

$48 \div 6 = __$ $\quad$ $48 \div 8 = __$

Help students achieve mastery. Keep record of which facts have been mastered and of the amount of time which is required to answer a fact test.

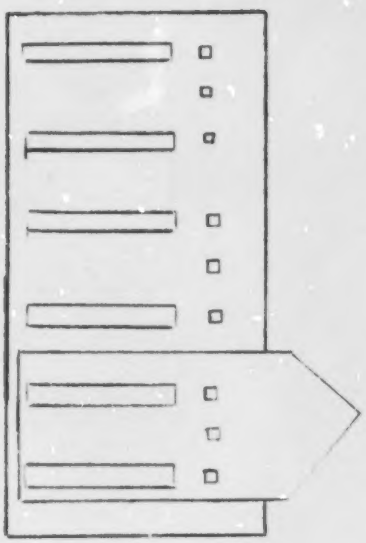
E-95 **Introduction to division by 2-digit divisors (Subtraction model with manipulatives):**

The repeated subtraction model using manipulatives is a good introduction to 2-digit divisors. The partitioning model gets unwieldy at this point. Use repeated subtraction just a short time -- to introduce the concept.

(Most texts use divisors such as 20, 30, 40, etc. (multiples of ten) to introduce this skill, but divisors such as 21, 32, etc.

can also be used. Their use sometimes help lessen confusion in minds of some students such as those who think the quotient for 20/80 is 40. Of course, selecting problems which don't require renaming in the subtractions also makes this introductory work easier. Example: 23/69 is easier than 17/51.)

Step 1

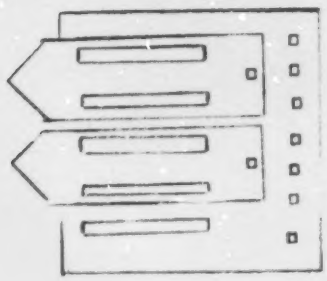


How many sets of 23 can you make?

(The model is repeated subtraction -- finding the number of sets. The thinking becomes that of finding the missing factor.
____ X 23 = 69 or
____ X 23 < 69)

```
23 / 69      or     23 / 69
-23                 -69   3 X 23
 46
-23
 23
-23
```

Remainders



```
    2 r 17
21 / 59
   -21
    38
   -21
    17
```

How large is the remainder? Is it larger than the divisor? than the quotient?

"Table of Facts"

```
    7
21 / 155
   -147
      8

    5
21 / 112
   -105
      7
```

1 X 21 = 21
2 X 21 = 42
3 X 21 = 63
4 X 21 = 84
5 X 21 = 105
6 X 21 = 126
7 X 21 = 147
8 X 21 = 168
9 X 21 = 189

Earlier, when students began division by 1-digit divisors with the subtraction model, they used multiplication facts to shorten the work. Instead of subtracting sets one at a time, they used multiplication facts (missing factor) to decide how many sets to subtract.

A "table of facts" for a particular 2-digit divisor will help students find the missing factor and lead them quickly away from reliance on manipulatives. Teacher can help students make such a table, then give them a number of problems which use the same divisor.

Checking

$$\begin{array}{r} 7 \\ 21 \overline{)\,155} \\ -147 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 21 \\ \times\ 7 \\ \hline 147 \\ +\ \ 8 \\ \hline 155 \end{array}$$

Use manipulatives if needed to review the checking procedure.

> Note: Rounding numbers to the nearest ten (Numeration Objective C-50) should be reviewed or taught before the next step.

Trial quotients:

$31 \overline{)\,155}$

Let's see if we can decide on a quotient without making a table of facts and without using blocks. What number should we <u>try</u>? (The word trial comes from try.) What do you know about these numbers that would help you guess which number to try?

(31 is close to 30, so think $150 \div 30$ in order to estimate.)

_____ X 30 = 150

Is it easier to think 15 tens ÷ 3 tens?

$22 \overline{)\,134}$

In this one, 2 is close to 20, so think $130 \div 20$. 130 is not a multiple of 20. What multiple of 20 could be subtracted? Would it help to count by 20s? (20, 40, 60, ...)

_____ X 20 < 130

Is it easier to think $13 \div 2$?

Sometimes the first guess (trial quotient) doesn't work. $28 \div 7 = 4$, but $4 \times 73 > 285$.

What should we do if the first try doesn't work?

Sometimes your second guess doesn't work either. What would you do then?
(For hesitant students, point out that there are only 10 numbers to guess. Eventually we can find the one which will work. We could even subtract one set at a time.)

Note: Two different models have been used throughout this paper: (1) the subtractive model which is used to find the number of sets and (2) the partitioning model which is used to find the number in each set. Both are useful at various points in development of the division algorithm. Both become unwieldy to model at this point (but can be referred to in discussion). Estimation must and should be the basic procedure now.

E-97
E-99 Introduction to division by 2-digit divisors for 2-digit quotients:

31 / 279

31 / 299

31 / 300

31 / 309

31 / 310

Give students series of problems such as these. (2-digit divisor with quotient of 9, then quotient of 9 plus remainder, and finally quotient of 10) Tell the students we are ready to discuss problems with 2-digit quotients.

Note: Multiplication-Division Objectives C41-46 (Multiplication by multiples of tens and hundreds) should be reviewed before the next step. (See Multiplication page 13.)

2-digit divisors, 2-digit quotients which are multiples of 10 (Missing factor concept):

31/310	31/620	31/930	10 X 31 = 310, so 310 ÷ 31 = 10.
44/440	44/880	44/1320	What number times 31 = 620?

Give students pairs or groups of related problems which will help them estimate quotients which are multiples of ten.

E-100 Estimating range of the quotient:

5/150

1 X 5 = 5
10 X 5 = 50
100 X 5 = 500

Let's use these multiplication sentences to decide whether the quotient will be between 1 and 10 or between 10 and 100. How many digits in the quotient?

7/29

1 X 7 = 7
10 X 7 = 70
100 X 7 = 700

How many digits in the quotient?

2/346

1 X 2 = 2
10 X 2 = 20
100 X 2 = 200

Continue same line of questioning.

5/150

1 X 5 = 5
10 X 5 = 50
100 X 5 = 500

31/64

1 X 31 = 31
10 X 31 = 310
100 X 31 = 3100

42/568

1 X 42 = 42
10 X 42 = 420
100 X 42 = 4200

2-digit divisors, 2-digit quotients which are not multiples of 10:

31/682

Let's work on this problem. How many digits in the quotient? (two) The answer will be between 10 and 100. Let's try a multiple of 10 such as 10, 20, 30, etc.

10 X 31 = 310
20 X 31 = 620
30 X 31 = 930

Step 1

(a)
$$\begin{array}{r} 31\overline{)682} \\ 620 \\ \hline 62 \end{array} \qquad 20 \times 31$$

(b)
$$\begin{array}{r} 20 \\ 31\overline{)682} \\ 620 \\ \hline 62 \end{array}$$

(c)
$$\begin{array}{r} 20 \\ 31\overline{)682} \\ 620 \\ \hline 62 \end{array}$$

Depending on the students' previous instruction, the teacher can choose one of these methods (a), (b), or (c) for recording step 1.

It is urged that the zeros be used briefly to aid in the estimation process.

Step 2

(a)
$$\begin{array}{r} 31\overline{)682} \\ 620 \\ \hline 62 \\ 62 \\ \hline 0 \end{array} \qquad \begin{array}{l} 20 \times 31 \\ \\ \\ \underline{\ 2} \times 31 \\ 22 \end{array}$$

(b)
$$\begin{array}{r} 22 \\ 2 \\ 20 \\ 31\overline{)682} \\ 620 \\ \hline 62 \\ 62 \\ \hline 0 \end{array}$$

(c)
$$\begin{array}{r} 22 \\ 31\overline{)682} \\ 620 \\ \hline 62 \\ 62 \\ \hline 0 \end{array}$$

We need to divide again for the ones digit.

(Use the method for recording step 2 which corresponds to the one used for step 1.)

Clearly, we want to move

$$\begin{array}{r} 22 \\ 31\overline{)682} \\ 620 \\ \hline 62 \\ 62 \\ \hline 0 \end{array} \quad \text{to} \quad \begin{array}{r} 22 \\ 31\overline{)682} \\ 62 \\ \hline 62 \\ 62 \\ \hline 0 \end{array}$$

as soon as possible. The following suggestions outline that process.

Note: Review procedure for estimating a trial quotient (Division page 27) before the next step.

Use of trial quotients for 2-digit divisors, 2-digit quotients:

$29\overline{)1218}$

With large numbers especially it is helpful to organize the work into steps.

First: Estimate the range of the quotient. Will the quotient be as large as 10? (yes) as 100? (no) How many digits will it have? (two)

Therefore, second: Divide for the tens digit. Decide what digit to try in tens place (trial quotient). The thinking is the same as that which was done for problems with smaller numbers. In this case 29 is close to 30 or 3 tens. 1218 is close to 12 hundreds. Think $12 \div 3$. Will 4 in tens place work? 4 in tens place means 40. Is $40 \times 29 < 1218$?

```
    40  <-- optional
29/1218
   1160  <-- optional

   etc.
```

Third: Subtract.

Fourth: Continue with the same steps which are used for problems with smaller numbers. i.e. Divide again to find the ones digit of the quotient.

Rational Numbers

There are many ways to interpret fractional numbers; each real-life application requires a certain interpretation, so children need to learn all the meanings eventually.

Some of these interpretations include fractions as measures of parts of regions, segments, and three-dimensional containers; fractions as part of a set of objects; fractions as ratios; fractions as quotients; and fractions as equivalent classes.

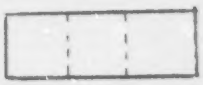
The following guide presents a series of <u>suggested</u> sequential step and strategies for teaching fraction concepts and algorithms. They are based on the manipulative approach, giving the student opportunities to work with models. These models help the student with discovery and visualization of fraction concepts. Although students do eventually need to learn all the interpretations of fractions, this guide works primarily with the measurement ideas (separating a single object into equal parts) and occasionally with the concept of separating or dividing a set of objects into equal shares or groups.

Throughout, the concept of <u>one</u> is stressed. The entire relationship is between the whole (which we call one) and parts of the whole. Often, difficulties can be avoided by asking students to focus on the whole set to which the fraction is related.

A-1
B-6

Separate an object into halves, thirds, fourths and name the parts:

"Johnny, here is a candy bar. Will you divide it with Tyrone so that each of you can have a fair share. What are each of these parts called."

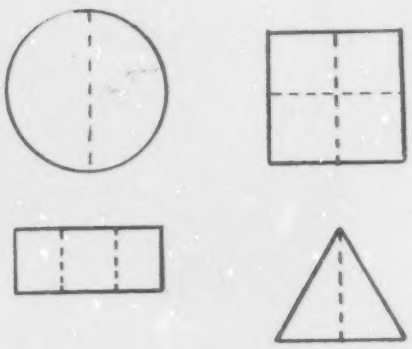


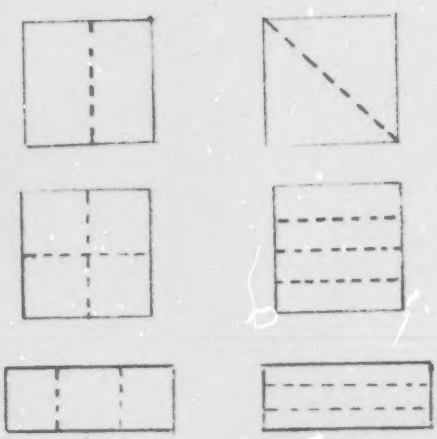
"Shawntelle, will you share this one with Mary and Marcus. What are each of these parts called?"

"If I divide this candy for you, which piece do you want. Why? Would you each get a fair share? These parts are not called thirds. The parts must be equal in size, and we must use the whole bar."

Another suggested activity:

Fold paper shapes (circles, squares, rectangles, triangles) into equal parts. Give a fraction name to each part.





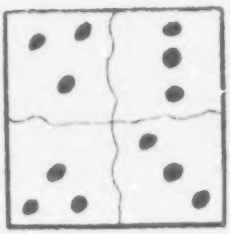
"How many different ways can you fold your shapes to show halves, fourths, and thirds?"

Recognize that the size of the fractional part depends on the size of the whole:

"Shade $\frac{1}{2}$ of each of the ellipses. What do you notice about the size of those halves? Why are they different?"

A-1
B-6

Separate a set of objects into halves, thirds, fourths and name the parts:



"Mary, let's share these 12 M and M's with Susan, Shawnetta, and Linda. How many will each of you get? What part of the whole set is that?"

(one-fourth)

Note: It is important for the students to notice the relationship of the fraction to the whole set; it is the same relationship as existed to the one object which they divided before. Concept of one or the whole is very important in work with fractions. With fractions, the relationship is important.

B-6 Write fractional notation:

"Willis, you're sharing this pizza with Jason and Tom. How many pieces do you need?" (3) "We write that this way $\frac{}{3}$ to show that there are three parts in the whole pizza. How many pieces will you get, Jason?" (1) "We write $\frac{1}{3}$ to show that your share is one-third. We read this $\left(\frac{1}{3}\right)$ as one-third."

Name orally and in writing fractions which are not unit fractions:

"Look at this circle. How many parts does it have?" (3)

"How many parts are shaded?" (1)

"Remember, we write $\frac{1}{3}$ and say that one-third is shaded."

"How many parts are not shaded?" (2)

"Since two out of three are not shaded, we say two-thirds is not shaded. We write $\frac{2}{3}$."

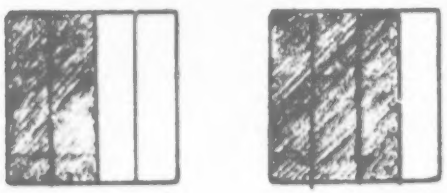
$\frac{2}{3}$ numerator
denominator

"Let's talk about what the numerals in a fraction stand for. The bottom numeral, the denominator, tells the total number of equal parts of the whole. The top numeral, the numerator, tells the number of equal parts we are talking about."

Note: Although students should be exposed to the numerator, denominator terminology, mastery of that vocabulary is not high priority at this level.

NOTE: For the following objectives, the committee suggests using the fraction circles and fraction bars as manipulative devices for comparing the size of fractions.

Compare the size of two fractions with different numerators and the same denominators:

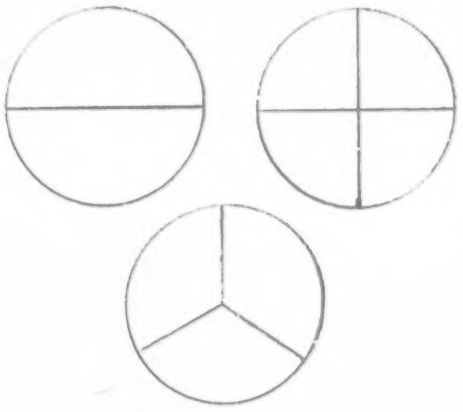


"Shade $\frac{2}{4}$ of the first square and $\frac{3}{4}$ of the second square. Which is larger $\frac{2}{4}$ or $\frac{3}{4}$?" $\left(\frac{3}{4}\right)$

"$\frac{2}{4}$ or $\frac{1}{4}$?" $\left(\frac{2}{4}\right)$

"How can we tell which fraction is larger? smaller?"

C-8 Compare the size of two fractions with the same numerator and different denominators:



"Look at the circles. Which is larger $\frac{1}{2}$ or $\frac{1}{4}$?" $\left(\frac{1}{2}\right)$

"$\frac{1}{4}$ or $\frac{1}{3}$?" $\left(\frac{1}{3}\right)$ "$\frac{2}{3}$ or $\frac{2}{4}$?" $\left(\frac{2}{3}\right)$

"Why?

How can we tell which fraction is larger? Smaller?"

C-9 Use < or > to compare the size of any two fractions: (both numerators and denominators may vary)

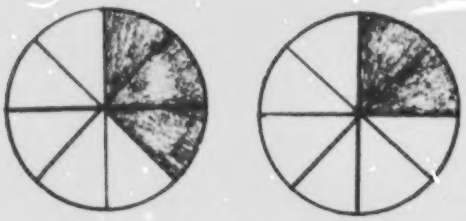
Note: Review meaning of the symbols < and >

"Shade $\frac{1}{2}$ of the top rectangle and $\frac{3}{4}$ of the bottom one. Which shaded part is larger?

What symbol would you use in this number sentence?

$\frac{1}{2}$ ◯ $\frac{3}{4}$ " (<)

D-15 Add like fractions: (informally)



"What fractional part of the first circle is shaded?" $\left(\frac{3}{8}\right)$

"Of the second circle?" $\left(\frac{2}{8}\right)$

"How much is shaded?" $\left(\frac{5}{8}\right)$

Note: The committee suggests using manipulative shapes (flannel, tagboard, vacu-form) to model this action.

Teacher writes:

$$\begin{array}{r} 3 \text{ eighths} \\ +2 \text{ eighths} \\ \hline 5 \text{ eighths} \end{array}$$

"What is the name of each part of this bar?" (one-fifth)

"Let's shade two-fifths. Now, two more fifths. How much is shaded?" (four-fifths)

Note: Sequence should include concrete models, then word form before developing the algorithm. This sequence helps prevent the error of adding denominators.

D-15 Add like fractions using an algorithm:

3 eighths + 3 eighths = 6 eighths

"You can see that three eighths plus three eighths equals six eighths. We write either

$\frac{3}{8} + \frac{3}{8} = \frac{6}{8}$ or

$$\begin{array}{r} \frac{3}{8} \\ +\ \frac{3}{8} \\ \hline \frac{6}{8} \end{array}$$

What rule* could we make for adding fractions with like denominators?" (If the fractions have the same denominator, write that denominator. Add the numerators of the fractions and write their sum as the numerator.)

*NOTE: The committee uses the word rule for want of a better synonym. We advocate having the children develop their own methods through the use of manipulatives and examples and then state those methods in words. The teacher is <u>not</u> to state a rule and teach from it.

Whenever students can perform an operation informally (orally), ask questions to make them aware of the method they are using. Use questions that lead them to state the algorithm in their own words. If they can discover the rules they are using, fine, but make sure they are stated. Don't assume that because students can give a correct answer they are using a correct algorithm. Very often students can get the right answer with a wrong method. By listening to their explanations, the teacher can determine what they are <u>thinking</u>.

D-16 Subtract like fractions: (informally)

"What fractional part of the first circle is shaded? $\left(\frac{3}{4}\right)$

If we remove one of these parts, what fractional part of the original circle is still shaded? $\left(\frac{2}{4}\right)$ "

Note: The committee suggests using manipulative shapes (flannel, tagboard, vacuform) to model this action.

Teacher writes:

$$\begin{array}{r} 3 \text{ fourths} \\ -\ 1 \text{ fourth} \\ \hline 2 \text{ fourths} \end{array}$$

Note: Sequence should include showing the concrete model, then recording the word form, before developing the algorithm.

D-16 Subtract like fractions using an algorithm:

3 fifths
-2 fifths
1 fifth

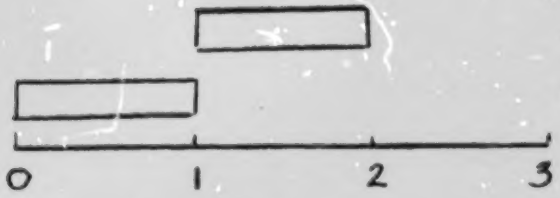
$$\begin{array}{r} \frac{3}{5} \\ -\ \frac{2}{5} \\ \hline \frac{1}{5} \end{array}$$

$\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$

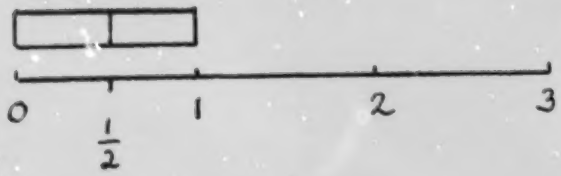
Proceed from word form to algorithm leading children to develop their own rules.

Provide practice with both the horizontal and vertical form.

D-19 Ordering fractions:

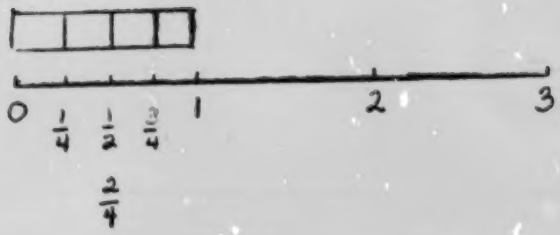


"Let's make a number-line to compare the size of fractions. One whole bar would measure the distance from 0 to 1. Two whole bars would measure from 0 to 2. If we laid one-half bar along the line from zero, where would its end point be?"



(Student points to spot. Teacher marks and labels number line.)

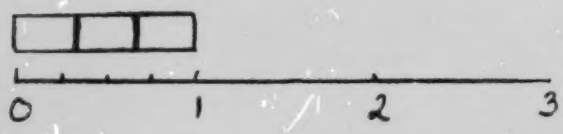
"If we laid one-fourth bar along the number line from zero, where would its end point be?"



(Student lays fourths bar along the number line, marks and labels the number line points for $\frac{1}{4}$ and $\frac{3}{4}$.)

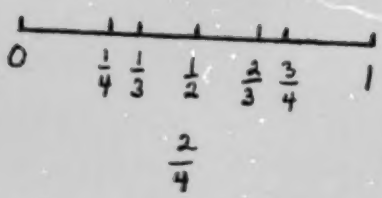
D-12,
D-17

"$\frac{2}{4}$ bar measured the same length as $\frac{1}{2}$ bar. We say that $\frac{2}{4}$ and $\frac{1}{2}$ are equivalent fractions.



"Suppose we separated the whole bar into thirds. Where would you locate $\frac{1}{3}$ and $\frac{2}{3}$ on the number line?"

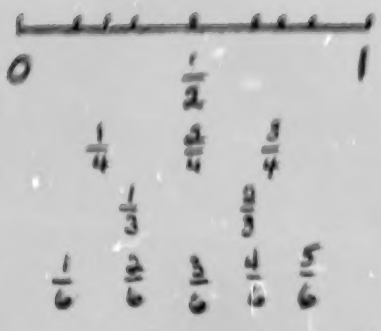
(Student indicates locations and marks them.)



"The number line between 0 and 1 looks like this now. Do the fractions appear to be in order by size?"

(Yes. $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{2}$ are in order; $\frac{1}{4} < \frac{1}{3}$, and $\frac{1}{3} < \frac{1}{2}$.

$\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$ are in order;.....)



"How can we tell where to mark sixths?"

(Continue same procedure with sixths bars.)

"Did you find any equivalent fractions?"

(Yes. $\frac{2}{6} = \frac{1}{3}$, $\frac{3}{6} = \frac{1}{2}$, and $\frac{4}{6} = \frac{2}{3}$.)

"Next, let's think about fractions like $\frac{2}{2}$, $\frac{4}{4}$ and $\frac{3}{3}$."

Recognize fractions equal to 1:

"What is the name of each fractional part in the first region?" (fourth)

Teacher writes $\frac{}{4}$.

"How many fourths are in the region?" (four)

Teacher writes $\frac{4}{4}$.

Repeat questioning for other regions.

"Could we say $\frac{4}{4} = 1$?

or $\frac{3}{3} = 1$? or $\frac{6}{6} = 1$?"

"Without making a picture, can you write another fraction which is equal to 1?"

"So $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} \ldots$

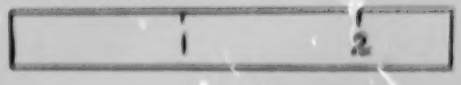
When the numerator and denominator are the same, the fraction equals one."

Know Meaning of Mixed Numbers:

"What name shall we give to these two whole regions and half of another region?"

(two and one half)

"This idea is written $2\frac{1}{2}$. It is called a mixed number because it mixes a whole number with a fraction."



This length measures $2\frac{1}{2}$ inches. It is 2 inches plus another half inch."

D-18 Identify fractions ≥ 1:

"How many fourths are there?" (seven)

"Write the fraction."

(Student writes $\frac{7}{4}$)

"There are enough parts to make one whole. This time there are some fourths left which will not make another whole. $\frac{7}{4}$ is called an improper fraction because it is greater than 1.

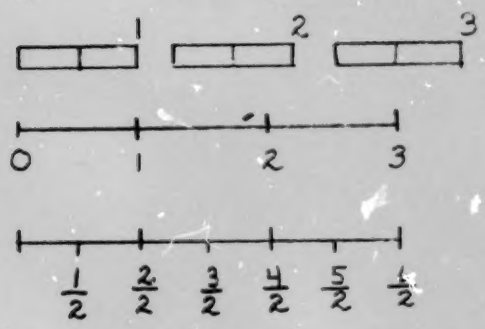
"$\frac{4}{4}$ is also an improper fraction. It is equal to 1."

"$\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$, $\frac{8}{4}$, $\frac{9}{4}$ are also improper fractions."

"Which of these are improper?"

$\frac{3}{5}$, $\frac{7}{5}$, $\frac{12}{6}$, $\frac{8}{3}$, $\frac{3}{4}$, $\frac{8}{8}$

Note: These ideas could also be illustrated with a number line which could be developed by putting a set of fraction bars in series.

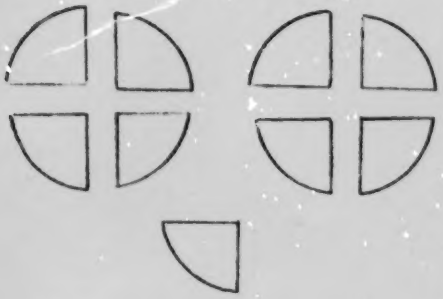


"Fractions which have a value less than 1 are called proper fractions.

$\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{5}$ are proper fractions."

D-21
F-42

Change Improper Fractions to Mixed Numbers or Whole Numbers:

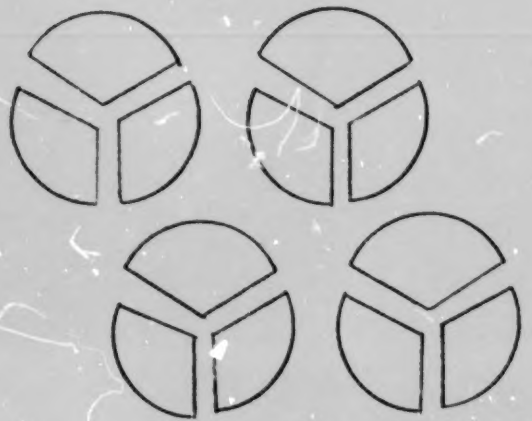


"What fraction could be used to name this model?" $\left(\frac{9}{4}\right)$

"If you rejoined these parts, how could you name the model with a mixed number?" $\left(2\frac{1}{4}\right)$

$\frac{9}{4} = \square$

"So does $\frac{9}{4} = 2\frac{1}{4}$?" (Yes)



"$\frac{12}{3}$ equals how many wholes?" (4

"Is that a mixed number?"

(No, there weren't any thirds left after we made wholes.)

"Improper fractions can be changed to whole numbers or mixed numbers.

$\frac{12}{3} = \square$

"Let's try some more examples and see if we can develop a rule for doing this."

(Rule: Look at the denominator of the fraction.

Example: for fourths, find number of groups of $\frac{4}{4}$ there are in the fraction. That number of groups is the whole number. If there are any more fourths, like $\frac{1}{4}$, $\frac{2}{4}$, or $\frac{3}{4}$, that number is the fraction part of the mixed number).

Note: A later stage of thinking will have the student divide the numerator by the denominator.

D-22
F-43

Change Mixed Numbers and Whole Numbers to Improper Fractions:

$2\frac{3}{4} = \frac{\square}{4}$

"Let's find a way to change mixed numbers and whole numbers to fractions. This picture shows $2\frac{3}{4}$. How many fourths is that altogether?" (eleven)

"Remember, we write that as $\frac{11}{4}$?" (Yes)

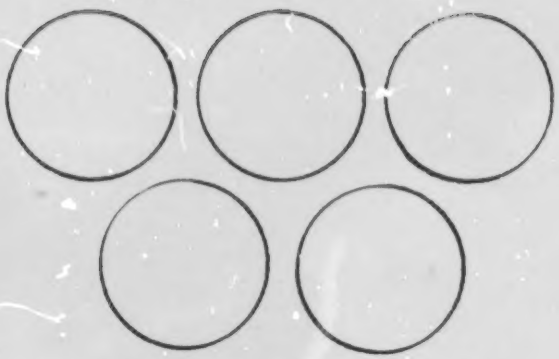
$\frac{4}{4} + \frac{4}{4} + \frac{3}{4} = \frac{11}{4}$

"We made $\frac{4}{4}$ from each whole circle so --

$\frac{4}{4} + \frac{4}{4} + \frac{3}{4} = \frac{11}{4}$ "

"Can we make a rule for this?"

Rule: Look at the denominator of the fraction. For example, if it is fourths, add the number of $\frac{4}{4}$ which is indicated by the whole number. Then add the fractional part of the mixed number.



$5 = \frac{\square}{4}$

"Remember, that some improper fractions become whole numbers, not mixed numbers. How could we change a whole number to an improper fraction?"

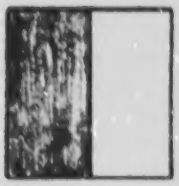
"Let's change 5 to fourths."

(Draw lines or cut)

"So 5 = $\frac{20}{4}$. Would our method still work?"

(Yes, but we can just multiply the whole number 5 by the denominator 4. Record this product as the numerator, and record the denominator. There is no fraction to add for this type of example.)

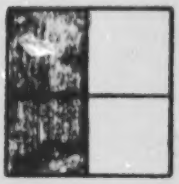
E-24 Rename fraction in
E-25 higher terms:



"How much of this square is shaded?" $\frac{1}{2}$

"If we fold the square like this, how many parts do we have?" (4)

Fold or draw:



"What could we call the shaded part now?" $\left(\frac{2}{4}\right)$

Fold or draw:

"What could we call the shaded part now?" $\left(\frac{4}{8}\right)$

"Does $\frac{1}{2} = \frac{2}{4}$ and also $\frac{4}{8}$?"

"Let's look at the idea again another way.

"How much of this circle is shaded?" $\left(\frac{2}{3}\right)$

Draw in dotted lines.

"Let's make two pieces from each part. How many parts are there now?" (6)

"How many are shaded?" (4)

"What fraction names the shaded part?" $\left(\frac{4}{6}\right)$

"So does $\frac{2}{3} = \frac{4}{6}$?" (Yes)

Draw in dotted lines.

"If we start again with $\frac{2}{3}$ shaded and make three pieces from each part, what fraction would be shaded?" $\left(\frac{6}{9}\right)$

"So does $\frac{2}{3} = \frac{6}{9}$?" (Yes)

Fraction	Pieces from each part	Equivalent Fraction
$\frac{2}{3}$	2	$\frac{4}{6}$
$\frac{2}{3}$	3	$\frac{6}{9}$
$\frac{2}{3}$	4	$\frac{8}{12}$
$\frac{2}{3}$	5	$\frac{10}{15}$

"Let's make a table to show what we've found."

"What would be next?"

(4 and $\frac{8}{12}$, then

5 and $\frac{10}{15}$)

"Do you see a pattern?"

(We multiplied the numerator and the denominator by the same number, and we found equivalent fractions.

$\frac{2 \times 2}{2 \times 3} = \frac{4}{6}$, so $\frac{2}{3} = \frac{4}{6}$

$\frac{3 \times 2}{3 \times 3} = \frac{6}{9}$, so $\frac{2}{3} = \frac{6}{9}$.)

Examples:

$\frac{2}{2} \times \frac{3}{4} = \frac{6}{8}$, so $\frac{3}{4} = \frac{6}{8}$

$\frac{3}{3} \times \frac{3}{4} = \frac{9}{12}$, so $\frac{3}{4} = \frac{9}{12}$

$\frac{5}{5} \times \frac{3}{4} = \frac{15}{20}$, so $\frac{3}{4} = \frac{15}{20}$

"When you multiply the numerator and the denominator by the same number you are really multiplying by 1 because $\frac{2 \times 3}{2 \times 4}$ is equivalent to multiplying $\frac{2}{2} \times \frac{3}{4}$, and $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}$ etc. are fractional forms equivalent to 1. Multiplying a number by 1 does not change its value. Multiplying

Multiplying a fraction by $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}$, etc. changes

its form, but not its value."

$\frac{1}{2} = \frac{\square}{10}$ $\quad \frac{7}{8} = \frac{\square}{16}$

$\frac{1}{5} = \frac{3}{\square}$ $\quad \frac{3}{8} = \frac{6}{\square}$

"Here are some problems. You will need to find a fractional form of 1 first.

Example:

$\frac{1}{2} = \frac{\square}{10}$.

Think: what number times 2 = 10."

E-24 **Rename fraction in lower terms:**

$\frac{1}{3} = \frac{2}{6} = \square = \frac{4}{12} = \square = \frac{6}{18}$

$\frac{3}{4} = \square = \frac{9}{12} = \frac{12}{16} = \square = \frac{18}{24}$

"What fractions are missing from these sets of equivalent fractions?"

$\square = \frac{2}{10} = \frac{3}{15} = \frac{4}{20}$

$\square = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$

$\square = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$

$\square = \frac{6}{10} = \frac{9}{15} = \frac{12}{20}$

"How did you do these?"

$\square = \frac{6}{8}$

$\square = \frac{10}{15}$

$\square = \frac{20}{40}$

"Try to find a fraction which is equivalent (equal in value) to each of these using smaller numbers (lower terms).

"How did you do these?"

Note: If student has found answers by patterning, he may have responses such as $\frac{4}{6}$, $\frac{6}{9}$, or $\frac{8}{12}$ for $\frac{10}{15}$.

Example:

Think $\frac{\square}{5} \times \frac{3}{4} = \frac{\square}{20}$, so $\frac{3}{4} = \frac{15}{20}$

"Remember, we changed fractions to higher terms by multiplying the numerator and denominator by the same factor. Could we change fractions to lower terms by finding factors?" (Yes)

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{15}{25}$

"Can you name some factors of 15?" (5, 3)

"Can you name some factors of 25?" (5, 5)

"What factor does 15 have in common with 25?" (5)

"So we can write:"

$$\frac{\square 5}{\square 5} \times \frac{\square}{\square} = \frac{15}{25}$$

$$\frac{\square 5}{\square 5} \times \frac{\square 3}{\square 5} = \frac{15}{25}$$

"Now, what are the missing factors?" (3,5) Teacher writes these factors in the boxes and says "So $\frac{15}{25} = \frac{3}{5}$ $\frac{3}{5}$ is in lower terms than $\frac{15}{25}$."

Example:

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{12}{16}$

"What numbers are factors of both 12 and 16?" (2 and 4)

$\frac{\boxed{2}}{\boxed{2}} \times \frac{\square}{\square} = \frac{12}{16}$,

"Let's try each one to see what happens."

"So does $\frac{12}{16} = \frac{6}{8}$?" (Yes)

$\frac{\boxed{4}}{\boxed{4}} \times \frac{\square}{\square} = \frac{12}{16}$

"And does $\frac{12}{16} = \frac{3}{4}$ also?" (Yes)

$\frac{3}{4} = \frac{6}{8} = \quad = \frac{12}{16}$

"We could put them in a table like this.

"Both $\frac{6}{8}$ and $\frac{3}{4}$ are in lower terms than $\frac{12}{16}$. Which is in lowest terms?" $\left(\frac{3}{4}\right)$

"If you had renamed $\frac{12}{16}$ as $\frac{6}{8}$, you could then have renamed $\frac{6}{8}$ as $\frac{3}{4}$ because 6 and 8 have a common factor, 2. $\frac{3}{4}$ is the simplest name for $\frac{12}{16}$ because 3 and 4 do not have a common factor."

$\frac{\boxed{2}}{\boxed{2}} \times \frac{\square}{\square} = \frac{6}{8}$

F-35 **Use the G.C.F. (Greatest Common Factor) to change fractions to lowest terms:**

"What numbers are factors of 12 and also of 18?"

(2, 3, and 6)

"2, 3, and 6 are all factors which are common to 12 and 18.. 6 is the greatest number from the set 2, 3, and 6. We say it is the greatest common factor. You can find the simplest name (lowest terms fraction) for $\frac{12}{18}$ by using the common factors in any of these ways:

$\frac{12}{18} = \frac{2}{2} \times \frac{\square}{\square}$; then $\frac{6}{9} = \frac{3}{3} \times \frac{\square}{\square}$ (1) using $\frac{2}{2}$, then $\frac{3}{3}$.

$\frac{12}{18} = \frac{3}{3} \times \frac{\square}{\square}$; then $\frac{4}{6} = \frac{2}{2} \times \frac{\square}{\square}$ (2) using $\frac{3}{3}$, then $\frac{2}{2}$.

$\frac{12}{18} = \frac{6}{6} \times \frac{\square}{\square}$ (3) using $\frac{6}{6}$.

"Each method is correct, but using the G.C.F. is the fastest way."

Use division notation to rename fractions in lowest terms:

$\frac{24}{30} = \frac{6}{6} \times \frac{\square}{\square}$

$\frac{24}{30} \div \frac{6}{6} = \frac{\square}{\square}$

"Do you think we could also think of these problems as division examples?"

Express whole number as a fraction with denominator of 1 (lowest terms):

$\frac{20}{4} = \frac{\square}{\square}$

$\frac{24}{8} = \frac{\square}{\square}$

$\frac{30}{5} = \frac{\square}{\square}$

"Let's try these examples. They can all be changed to lowest terms with a denominator of 1.

$\frac{20}{4} \div \frac{4}{4} = \frac{5}{1}$."

"Remember that you could change some improper fractions to whole numbers.

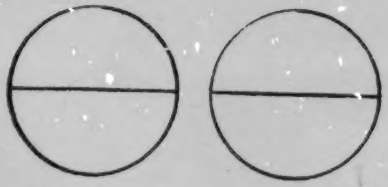
$\frac{20}{4} = 5$."

"5 can be written $\frac{5}{1}$."

"Let's check this idea with some models."

$1 = \frac{2}{2}$

"1 has been divided into 2 parts. 2 in the denominator names the parts."



$2 = \frac{4}{2}$

$3 = \frac{12}{4}$

"3 wholes have been separated into 4 parts each, so the denominator is 4.

$2 = \frac{2}{\square}$

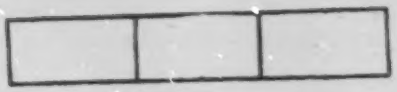
"What would the denominator be here?"

(1) Yes, each whole has not been divided. Therefore, the denominator is 1."

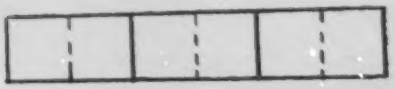
Any whole number can be re-written with a denominator of 1.

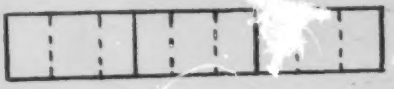
"$2 = \frac{2}{1}$, $4 = \frac{4}{1}$, $7 = \frac{7}{1}$, etc."

E-26 **Find the L.C.D. Lowest (Least) Common Denominator of a pair of fractions:**

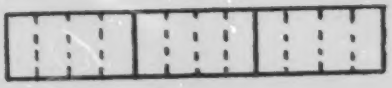


"This fraction bar has been separated into thirds. The thirds could be changed to sixths,





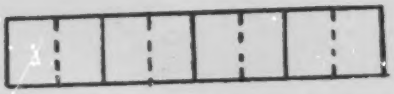
or ninths,

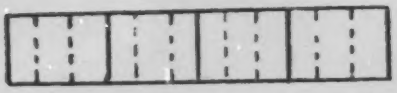


or twelfths,

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18} = \frac{7}{21} = \frac{8}{24} = \ldots$$

"This fraction bar has been separated into fourths. The fourths could be changed to eighths,





or twelfths,

etc.

$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} = \ldots$

"Later we will need to change two fractions to the same kind of parts. We will change the fractions to equivalent forms so both will have the same denominator."

"Can you find a denominator that will work for both thirds and fourths?" (12, 24)

"Yes, either will work but 12 is the lower one. We say 12 is the lowest (least) common denominator (L.C.D.) for thirds and fourths."

Note: Some authors also use the terminology, L.C.M., for the least common multiple.

"Let's try a few more fractions, then develop a rule or method to use for finding the L.C.D."

Examples:

$\frac{1}{4} =$

"What denominators will work for fourths?" (4, 8, 12, 16, 20, 24, ...)

$\frac{1}{6} =$

"What denominators will work for sixths?" (6, 12, 18, 24 32, ...)"

"What did we call those sets of numbers?" (multiples of 4, multiples of 6)
"What denominators are common for fourths and sixths?" (12, 24, ...)
"Which is the L.C.D.?"

Example:

$\frac{3}{4}$

$\frac{1}{8}$

"What are the multiples of 4?" (4, 8, 12, 16, ...)
"of 8?" (8, 16, 24, 32, ...)
"Which multiples are common to both?" (8, 16, 24)
"Did you notice that 8 is the least common denominator? 16 is not the least one. Also, notice 8 is one of the original denominators."

"Would 4 work?" (no)
"Why not?"

"In which list of multiples did you find the L.C.D. with the least number of tries?" (the 8 which is the larger denominator and which represents the smaller parts).

"What rule can we write for steps in finding the L.C.D.?" (Make a list of multiples for each denominator. Pick the least number that is in both lists.)

A quicker method is to look at the larger denominator, then at each of its multiples. Test each to see if that number is also a multiple of the smaller denominator. The smallest common multiple is the one we want. (L.C.D.)

Note: Review adding like fractions. Mastery of these skills is necessary before proceeding to the next objective.

E-28 Add unlike fractions:

Note: The committee suggests using circle models for introducing this skill, because the whole (one) is easily recognizable.

"Kenny, here is part of a circle. What name do we give it?" $\left(\frac{1}{2}\right)$

Write $\frac{1}{2}$ on the board.

"Angela, here is another part of a circle. What name do we give it?" $\left(\frac{1}{4}\right)$

Write $\frac{1}{4}$ on the board.

"When we join these two parts what fractional part of a circle do we have?"

Children will probably have difficulty at this point.

$\frac{1}{2} + \frac{1}{4} = \frac{\square}{\square}$

"We want to join $\frac{1}{2}$ and $\frac{1}{4}$. We do that by addition." Write the (+) sign between the $\frac{1}{2}$ and $\frac{1}{4}$.

$\frac{1}{2} + \frac{1}{4} = \frac{\square}{\square}$

$\downarrow \quad \downarrow$

$\frac{2}{4} + \frac{1}{4} = \frac{\boxed{3}}{\boxed{4}}$

"We know how to add like fractions. What is different about this number sentence?" (Denominators are different)
"We need to change these fractions so they have common denominators. Then we can use our rule for adding like fractions."

"Is our answer, $\frac{3}{4}$, a name for the fractional part of the circle we modeled?" (Yes)

Note: Develop as many examples as necessary with models. Then proceed to the algorithm. It would be possible to teach simplification of answers here, or it may be delayed. (See Rational Numbers Objective F-36.)

Choose between sentence form and computation form:

$\frac{2}{3} + \frac{1}{4} = \frac{\square}{\square}$

$\frac{8}{12} + \frac{3}{12} = \frac{\square}{\square}$

$\frac{2}{3} = \frac{8}{12}$

$+ \frac{1}{4} = \frac{3}{12}$

"We can write problems either as number sentences like this or in computational form like this. When we need to change to common denominators, computational form is usually easier.

Note: Although students need to understand both sentence form and computational form for addition/subtraction of fractions, use of both forms interchangeably is confusing to some children. When the problems require changing fractions to higher terms, both forms have a similar boxlike appearance.

$$\boxed{\begin{array}{l}\frac{2}{3}+\frac{1}{4}=n\\ \frac{8}{12}+\frac{3}{12}=n\end{array}} \qquad \boxed{\begin{array}{l}\frac{2}{3}=\frac{8}{12}\\ \frac{1}{4}=\frac{3}{12}\\ \hline\end{array}}$$

Students may make perceptual errors and/or forget which form they are using.

Students usually find the computational form easier than sentence form. We recommend introducing both forms, then teaching students to change to the computational form for all addition and subtraction work.

Note: Review subtracting like fractions before proceeding to the next objective.

E-29
E-30

Subtract unlike fractions:

"Joy, what part of a circle is this?" $\left(\frac{7}{8}\right)$

Write $\frac{7}{8}$ on the board.

"Reggie, can you separate $\frac{2}{4}$ of the circle from the $\frac{7}{8}$?

Write $\frac{2}{4}$ on the board.

$$\frac{7}{8} - \frac{2}{4} = \frac{\square}{\square}$$

"We show the separation with this number sentence."

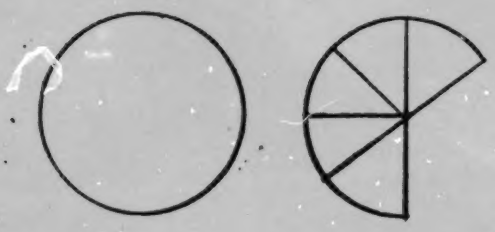
Computational form:

$$\begin{array}{r} \frac{7}{8} = \frac{7}{8} \\ -\ \frac{2}{4} = \frac{4}{8} \\ \hline \frac{3}{8} \end{array}$$

"Do we know how much of the circle is left? What do we have to do to solve this number sentence?" (Change to like fractions and then subtract.)

Note: As with addition of unlike fractions, develop as many examples as necessary with models and then proceed with algorithm. It would be possible to teach simplificatio of answers here or it may be delayed. (See Rational Numbers Objective F-37.)

Add two mixed numbers (like denominators):



"Write the numeral for this model." $\left(1\ \frac{5}{8}\right)$

"Write the numeral for this model." $\left(2\ \frac{1}{8}\right)$

$1\frac{5}{8} + 2\frac{1}{8} = \square$

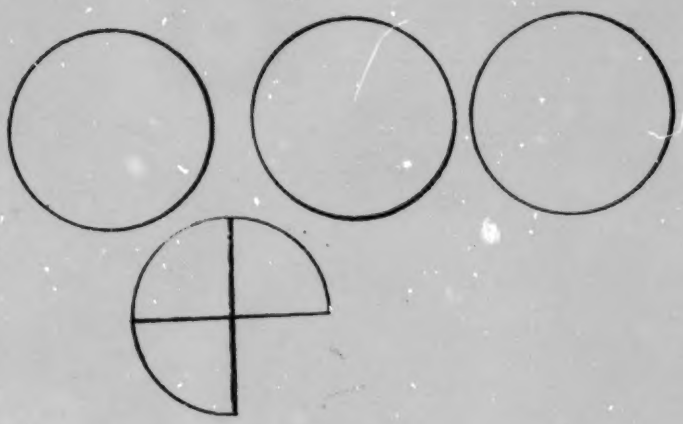
"How would you show the joining of these models?" (+)

Computational form:

$$\begin{array}{r} 1\frac{5}{8} \\ +2\frac{1}{8} \\ \hline 3\frac{6}{8} \end{array}$$

Note: It would be possible to teach simplification of answers here or it may be delayed. (See Rational Numbers Objective F-36)

Subtract mixed numbers (like denominators):



"Write the numeral for this model." $\left(3\frac{3}{4}\right)$

"Let's separate $1\frac{1}{4}$ from this model. We write this number sentence to show the action.

$$3\tfrac{3}{4} - 1\tfrac{1}{4} = \square$$

"How much is left?" $\left(2\frac{2}{4}\right)$

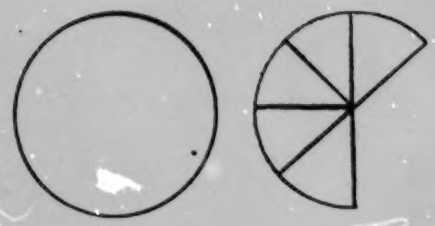
Computational form:

$$\begin{array}{r} 3\ \frac{3}{4} \\ -\ 1\ \frac{1}{4} \\ \hline 2\ \frac{2}{4} \end{array}$$

Note: It would be possible to teach simplification of answers here or it may be delayed. (See Rational Numbers Objective F-37.)

E-31 Add two mixed numbers (unlike denominators):

"Write the numeral for this model." $\left(2\frac{3}{4}\right)$



"Write the numeral for this model." $\left(1\frac{5}{8}\right)$

$$2\frac{3}{4} + 1\frac{5}{8} = \square$$

"How would we show the joining of these models?" (+)

Computational form:

$$\begin{array}{r} 2\frac{3}{4} = 2\frac{6}{8} \\ +\,1\frac{5}{8} = 1\frac{5}{8} \\ \hline 3\frac{11}{8} \end{array}$$

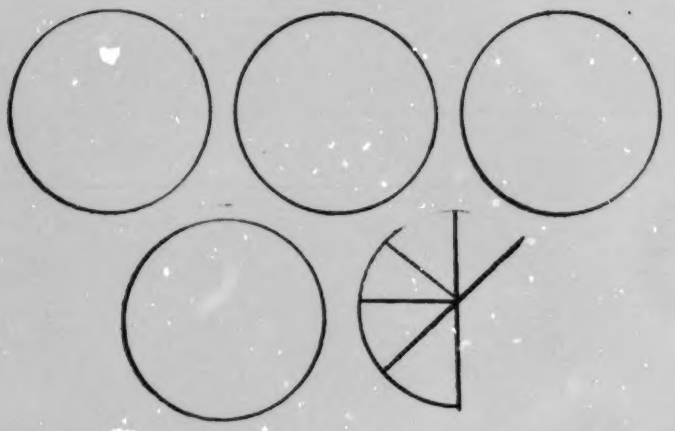
"How do we proceed with this problem?" (Change to like fractions and then add.)

Note: It is important to use the equal sign correctly.

Correct	Incorrec
$2\frac{3}{4} = 2\frac{6}{8}$	$2\frac{3}{4} = \frac{6}{8}$
$+\ 1\frac{6}{8} = 1\frac{6}{8}$	$+\ 1\frac{6}{8} = \frac{6}{8}$
$3\frac{12}{8}$	$3\ \frac{12}{8}$

It would be possible to teach simplification of answers here or it may be delayed. (See Rational Numbers Objective F-36.)

E-31 **Subtract mixed numbers (different denominators):**



"Write the numeral for this model." $\left(4\ \frac{5}{8}\right)$

"Let's separate $2\ \frac{3}{16}$ from this model. Will we have a problem doing this?" (Yes) "Why" (Model is separated into eighths and not sixteenths.)

$$4\ \frac{5}{8} - 2\ \frac{3}{16} = \square$$

"We write this number sentence to show the separation.

Computational form:

$$4\ \frac{5}{8} = 4\ \frac{10}{16}$$
$$-2\ \frac{3}{16} = 2\ \frac{3}{16}$$
$$2\ \frac{7}{16}$$

"Remember, we need to change to like fractions and then subtract."

Note: Use correct number sentence form here.

Correct	Incorrect
$4\ \frac{5}{8} = 4\ \frac{10}{16}$	$4\ \frac{5}{8} = \frac{10}{16}$
$-2\ \frac{3}{16} = 2\ \frac{3}{16}$	$-2\ \frac{3}{16} = \frac{3}{16}$
$2\ \frac{7}{16}$	$2 \quad \frac{7}{16}$

It would be possible to teach simplification of answers here or it may be delayed. (See Rational Numbers Objective F-37.)

F-36 **Add any two unlike fractions and name the sum in simplest terms:**

Note: Below are examples of different types of problems and answers when adding unlike fractions. Refer to the Rational Numbers Objectives listed after each example for the strategies for solving and simplifying the answer.

(1) $\frac{3}{10} = \frac{3}{10}$

$+ \frac{1}{2} = \frac{5}{10}$

$\frac{8}{10}$ or $\frac{4}{5}$

Objectives E-28, 30
F-35

(2) $\frac{11}{12} = \frac{11}{12}$

$+ \frac{3}{4} = \frac{9}{12}$

$\frac{20}{12}$ or $1\frac{8}{12}$ or $1\frac{2}{3}$

Objectives D-21
E-28, 30
F-35

Alternate way:

$\frac{11}{12} = \frac{11}{12}$

$+ \frac{3}{4} = \frac{9}{12}$

$\frac{20}{12}$ or $\frac{5}{3}$ or $1\frac{2}{3}$

(3) $12\frac{3}{10} = 12\frac{3}{10}$

$+ \quad 8\frac{1}{2} = 8\frac{5}{10}$

$20\frac{8}{10}$ or

$20\frac{4}{5}$

Objectives E-31
F-35

(4) $4\frac{5}{6} = 4\frac{5}{6}$

$+ \quad 1\frac{2}{3} = 1\frac{4}{6}$

$5\frac{9}{6}$

Objectives E-31
F-35

$5\frac{9}{6} = 5 + 1\frac{3}{6} = 6\frac{3}{6}$

or $6\frac{1}{2}$

Alternate way:

$5\frac{9}{6} = \frac{39}{6} = 6\frac{3}{6}$ or $6\frac{1}{2}$

F-36 Subtract unlike fractions (no renaming, "borrowing") and name the difference in simplest terms:

Note: Below are examples of different types of problems and answers when subtracting unlike fractions. Refer to the Rational Numbers Objectives listed after each example for the strategies for solving and simplifying the answer.

(1) $\frac{11}{12} = \frac{11}{12}$

$- \frac{3}{4} = \frac{9}{12}$

$\frac{2}{12}$ or $\frac{1}{6}$

Objectives E-29, 30
F-35

(2) $9\frac{5}{6} = 9\frac{5}{6}$

$- 4\frac{1}{2} = 4\frac{3}{6}$

$5\frac{2}{6}$ or $5\frac{1}{3}$

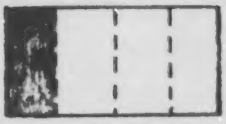
Objectives E-31
F-35

Subtract (like denominators) with renaming ("borrowing"):

From whole numbers

"Anita, this rectangle is 'one'. Can you separate $\frac{1}{4}$ from it?" (No)

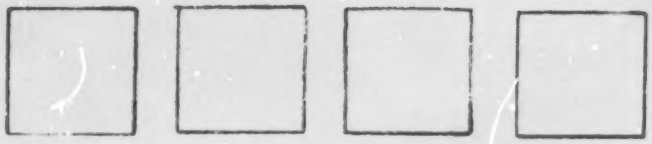
"Why not?" (Rectangle needs to be divided into fourths.)



"Let's draw dotted lines to show the fourths and shade in $\frac{1}{4}$. How much is left unshaded?" $\left(\frac{3}{4}\right)$

"So $1 - \frac{1}{4} = \frac{3}{4}$. Let's write that in computational form. Now can we rename the 1, so that we can subtract $\frac{1}{4}$ from it?" $\left(\frac{4}{4}\right)$

$$\begin{array}{rcl} 1 & = & \frac{4}{4} \\ -\ \frac{1}{4} & = & \frac{1}{4} \\ \hline & & \frac{3}{4} \end{array}$$



"Jesse, look at these 4 squares. Can you separate $\frac{2}{3}$ from them?" (Not without dividing into thirds).

"Do we need to divide each whole into thirds in order to separate $\frac{2}{3}$?"

(No, we only need to divide one whole.) "How would we name our model now?"

$\left(3\frac{3}{3}\right)$

"Can we separate $\frac{2}{3}$ from $3\frac{3}{3}$?"

(Yes) "We write the problem this way."

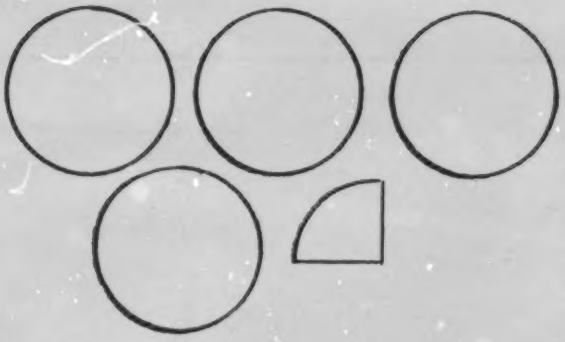
$$\begin{array}{r} 4 = 3\frac{3}{3} \\ -\ \frac{2}{3} = \frac{2}{3} \\ \hline 3\frac{1}{3} \end{array}$$

<u>Note</u>: The committee suggests physically removing the shaded parts from the model.

Use similar models and questioning to develop method for subtracting a mixed number from a whole number.

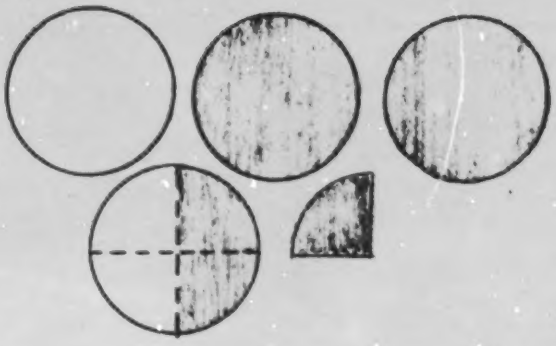
$$\begin{array}{r} 5 = 4\frac{6}{6} \\ -\ 2\frac{5}{6} = 2\frac{5}{6} \\ \hline 2\frac{1}{6} \end{array}$$

From mixed numbers



"Yolanda, what number names this model?" $\left(4\frac{1}{4}\right)$

"Let's separate $2\frac{3}{4}$ from it." (We can't.) "What do we have to do?" (Divide one of the wholes into fourths.)



"Now, how could you name this model?" $\left(3\frac{5}{4}\right)$

"Yes, $4\frac{1}{4}$ has been renamed.

$4\frac{1}{4} = 3 + \frac{4}{4} + \frac{1}{4}$, or $3\frac{5}{4}$.

Can we separate $2\frac{3}{4}$? (Yes)

Note: shading indicates removal.

"What is left?" $\left(1\frac{2}{4}\right)$

"We write the problem this way."

$$\begin{array}{rl} & {}^{3}\not{4}\frac{1}{4} = 3\frac{5}{4} \\ - & 2\frac{3}{4} = 2\frac{3}{4} \\ \hline & 1\frac{2}{4} \text{ or } 1\frac{1}{2} \end{array}$$

Note: Modeling the action here is important so that children can visualize the renaming ("borrowing"). A common mistake is to move over a 1 as is done in subtraction of whole numbers.

Incorrect:

$$\begin{array}{r} {}^{3}\not{4}\frac{1}{4} \\ -\ 2\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 7\frac{3}{8} = 6\frac{11}{8} \\ -\ \frac{7}{8} = \frac{7}{8} \\ \hline 6\frac{4}{8} \text{ or } 6\frac{1}{2} \end{array}$$

Use similar models and questioning to develop method for subtracting a fraction from a mixed number.

F-43 Subtract (unlike denominators) with renaming ("borrowing"):

$$\begin{array}{r} 7\frac{2}{3} = 7\frac{4}{6} = 6\frac{10}{6} \\ -\ 2\frac{5}{6} = 2\frac{5}{6} = 2\frac{5}{6} \\ \hline 4\frac{5}{6} \end{array}$$

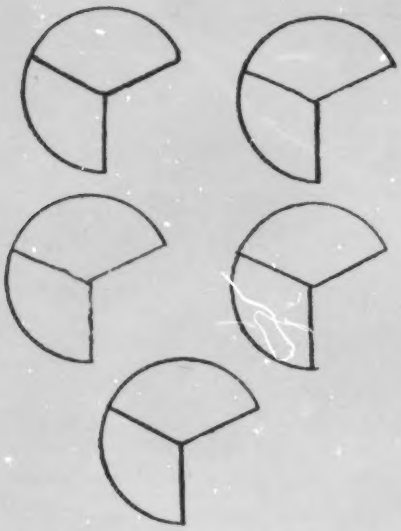
$$\begin{array}{r} 8\frac{1}{3} = 8\frac{2}{6} = 7\frac{8}{6} \\ -\ \frac{1}{2} = \frac{3}{6} = \frac{3}{6} \\ \hline 7\frac{5}{6} \end{array}$$

The problems at the left must be solved in two steps:

1) Change fractions to common denominators. (Refer to Rational Numbers Objective E-26 for review of this skill.)

2) Rename ("borrow") minuend. (Refer to Rational Numbers Objective E-26 and F-36 for review of this skill.)

Multiplication
Whole Number
x fractions:



$5 \times \frac{2}{3} = \frac{10}{3}$ or $3\frac{1}{3}$

"Five girls each ordered a pizza. Each girl ate $\frac{2}{3}$ of hers. How much pizza was eaten?"

"Could we write this as an addition problem?" (Yes)

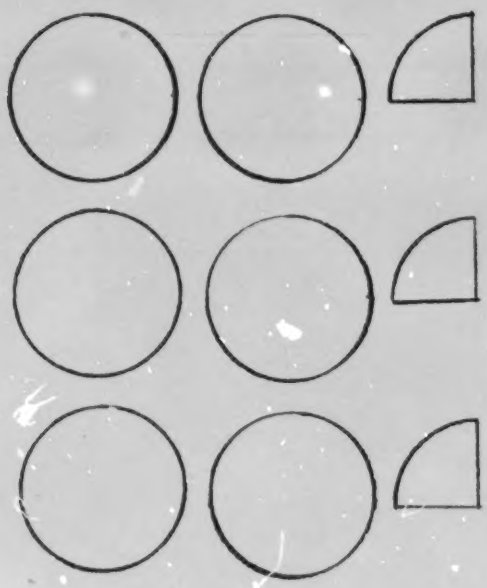
$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3} = \frac{10}{3}$

"Joining 5 groups of two-thirds is $\frac{10}{3}$. We can write that as a multiplication sentence."

Continue with other examples and questions leading children to develop rule: (Multiply whole number times the numerator and record the denominator.)

Note: The committee suggests that the teacher defer teaching fractions x whole number $\left(\frac{2}{3} \times 5\right)$ at this time because that concept is easier to model at a later stage.

Whole number
x mixed number:



"Three boys in a pie eating contest each ate $2\frac{1}{4}$ pies. How much pie did they eat?"

"Could we write this as an addition problem?" (Yes)

$$\begin{array}{r} 2\frac{1}{4} \\ 2\frac{1}{4} \\ +\ 2\frac{1}{4} \\ \hline 6\frac{3}{4} \end{array}$$

"We are joining 3 groups of two and one-fourth. Can you see that this is the same as joining 3 groups of two (6) and 3 groups of one-fourth $\left(\frac{3}{4}\right)$?"

$3 \times 2\frac{1}{4} = 6\frac{3}{4}$

"We can write that as a multiplication sentence."

Note: This skill uses the distributive property.

$3 \times 2\frac{1}{4} = (3 \times 2) + (3 \times \frac{1}{4})$

The committee suggests developing this intuitively. The teacher must decide whether he/she wants to develop a rule with her/his students or if he/she wants to defer development of this skill until after students have learned the rule for multiplying fraction times fraction.

Estimation to check reasonableness of products.

$8 \times 4 =$

$4 \times 4 =$

$2 \times 4 =$

"Let's complete this table. What's happening here?" (The products are getting smaller.)

"Are they all greater than 4?" (Yes)

$1 \times 4 =$

"Let's extend the table." (The product equals 4.)

$\frac{1}{2} \times 4 =$

"Now, what will happen?" (The product will be smaller than 4.)

$\frac{1}{4} \times 4 =$

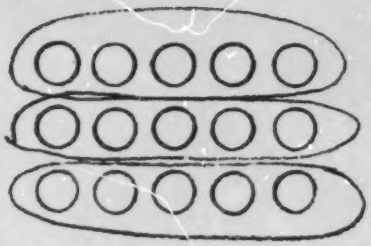
$\frac{1}{8} \times 4 =$

"What would you expect to happen if you extend the table?" (The products will get smaller.)

F-40 **Fraction times whole number; (Using the fraction of a set interpretation)**

Note: This real-life problem is presented using a different interpretation of fractions than is used in the rest of this guide. Under this interpretation, the whole (or one) is a set of objects, not a single object or region. The student must find a part of the whole set.

Later, we will reconsider this kind of problem under the other interpretation.



"Jason, Suzie, and Samantha had 15 cookies to share. How many would each get?"

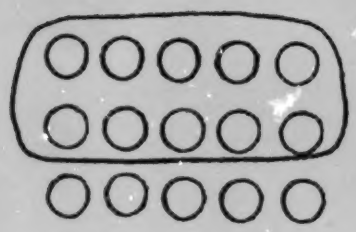
"What fraction would name each child's share?" $\left(\frac{1}{3}\right)$

"How many cookies would that be?" (5)

"What did you do?" (Divided 15 by 3.)

"So $\frac{1}{3}$ of 15 is 5." "Of" means "times", so we can write that as $\frac{1}{3} \times 15 = 5$.

$\frac{1}{3} \times 15 = 5$



$\frac{2}{3} \times 15 = 10$

"Suzie remembered that she didn't like that kind of cooky and gave her share to Samantha. How many cookies did Samantha get altogether? Would the answer be as much as 15?"

"What fraction would name Samantha's share now?" $\left(\frac{2}{3}\right)$

How many cookies would that be? (10) "So $\frac{2}{3}$ of 15 is 10. "Of" means "times" so we write $\frac{2}{3} \times 15 = 10$."

"What did you do this time?" (Divided 15 by 3 and multiplied by 2.)

Develop other examples leading children to verbalize rule: Divide whole number by denominator and multiply by numerator. (Take caution in selecting examples so that students are not led into trying something too difficult such as whole number not divisible by denominator.)

Problems of this type (finding a fraction of a set) occur frequently, s children need to learn how to solve them. Many children, however, think o them as division because they have to divide first. Sometimes they interpret the problem as $15 \div \frac{2}{3}$.

The teacher needs to be alert to the children's thinking. They need to b taught that they are multiplying by a fraction.

Probably a quick exposure to this line of thinking will be enough, because children will learn to multiply whole numbers by fractions in this more efficient way later:

$$\frac{2}{3} \times 15 = \frac{2}{3} \times \frac{15}{1}$$

Commutative principle applies to multiplication of fraction:

"How much is $\frac{1}{2}$ of 4?"

(2)

$4 \times \frac{1}{2} =$

"How much are 4 one-halves?" (2)

"Is $\frac{1}{2} \times 4$ equal to $4 \times \frac{1}{2}$?" (Yes, they both equal 2.) "We can turn the problem around and not change the answer. The commutative principle is true for multiplication with fractions just like it is with whole numbers."

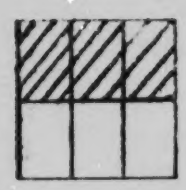
F-39 Fraction times fraction:

Note: The committee suggests using the fraction squares and transparencies for these examples. When using these, the students must be aware that these models of square regions represent the same whole.

$\frac{1}{2} \times \frac{1}{3} =$

"We're going to multiply a fraction by a fraction. Let's find out what $\frac{1}{2}$ of $\frac{1}{2}$ is. Here is a cardboard square. Color $\frac{1}{3}$ of it red."

"Here is a transparency of the same square. Show $\frac{1}{2}$ of it."



"Now let's lay the transparency on the cardboard square. Can you still see $\frac{1}{3}$ of it?"

(Yes) "Show me. Can you still see $\frac{1}{2}$ of it?"

(Yes) "Show me. Where is $\frac{1}{2}$ of $\frac{1}{3}$? What would we name that part?" $\left(\frac{1}{6}\right)$

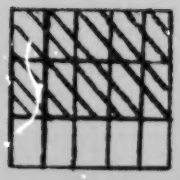
"So $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$."

"Let's try another example.

$\frac{2}{3} \times \frac{4}{5} =$

"Color $\frac{4}{5}$ of this cardboard square blue."

"Here is a transparency of the same square. Show $\frac{2}{3}$ of it."



"When we lay the transparency on the cardboard square, this portion (Point to the appropriate section) shows $\frac{2}{3}$ of $\frac{4}{5}$. What part of the whole square is that?" $\left(\frac{8}{15}\right)$. So we can say that $\frac{2}{3}$ of $\frac{4}{5}$, or $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$."

Continue writing and modeling other examples to lead children to develop rule: (Multiply numerator times numerator and denominator times denominator.)

F-44
F-45
F-48

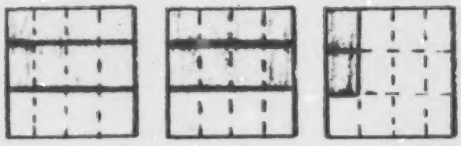
Find product of any two numbers from the sets of whole numbers, common fractions, and mixed numbers:

Note: Below are listed examples of the three kinds of problems in multiplication of fractions that we have not yet introduced. We will solve these problems by changing whole and mixed numbers to fractions and then by using the rule for multiplying fraction times fraction. Refer to Rational Numbers Objective F-39. The committee suggests modeling to prove only a few examples.

Fraction times mixed number

$\frac{2}{3} \times 2\frac{1}{4} = \square$

$\frac{2}{3} \times \frac{9}{4} = \frac{18}{12}$ or $1\frac{6}{12}$ or $1\frac{1}{2}$

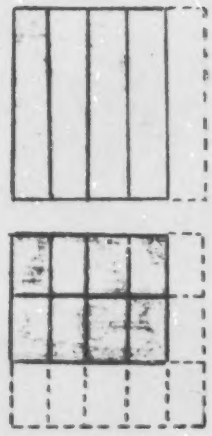


"Let's use a model to see if that answer is correct. How many twelfths are there?" (18) "If the twelfths were rearranged, what would the answer be in simplest terms?"

Mixed number times fraction

$1\frac{2}{3} \times \frac{4}{5} = \square$

$\frac{5}{3} \times \frac{4}{5} = \frac{20}{15} = 1\frac{5}{15}$ or $1\frac{1}{3}$

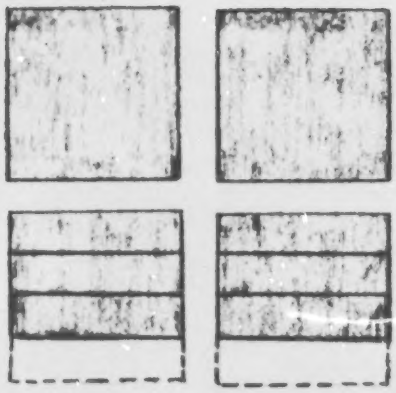


Prove by modeling.

Mixed number
times whole
number

$1\frac{3}{4} \times 2 = \square$

$\frac{7}{4} \times \frac{2}{1} = \frac{14}{4} = 3\frac{2}{4}$ or $3\frac{1}{2}$

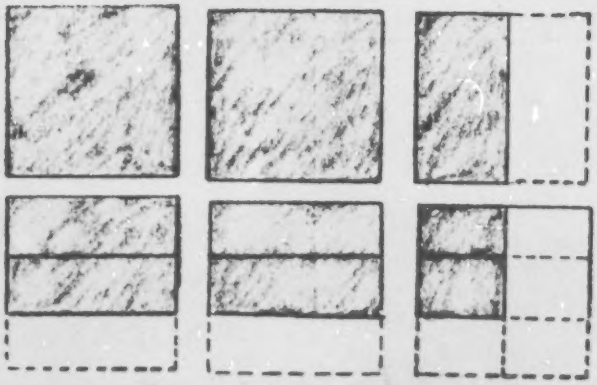


Prove by modeling.

Mixed number
times mixed
number

$1\frac{2}{3} \times 2\frac{1}{2} = \square$

$\frac{5}{3} \times \frac{5}{2} = \frac{25}{6} = 4\frac{1}{6}$

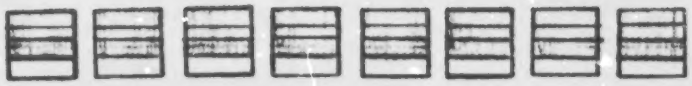


Prove by modeling.

Fraction times whole numbers

$\frac{3}{4} \times 8 = \square$

$\frac{3}{4} \times \frac{8}{1} = \frac{24}{4} = 6$



Prove by modeling. (The whole set is 1.)

The initial presentation of this skill used the interpretation of fractions as part of a set of objects because most real world applications for this skill came from that interpretation. Under that interpretation $\frac{3}{4}$ of 8 would be modeled:

(The whole set is 8.)

It is probably not important for the teacher to call the student's attention to these differences in interpretation, but the teacher should be aware of what the student is thinking, especially if he is having difficulty.

Using estimation to check reasonableness of products:

"When we multiply using fractions, as in whole numbers, we want to be sure our answer is reasonable. By estimating and recognizing limits, we can judge better the reasonableness of our answers."

$4 \times 0 = 0$

"When we multiply 4 x 0, what do we get?" (0)

$4 \times 1 = 4$

"If we multiply 4 x 1, what do we have?" (4)

Whole number times a fraction

$4 \times 0 = 0$

$4 \times \frac{2}{5} =$

$4 \times 1 = 4$

"By making a table, what do we see would be the range of answers we could get when we multiply 4 times a fraction?"
(Between 0 and 4)

"We can use tables to make other estimations."

Whole number times a mixed number

$3 \times 2 = 6$

$3 \times 2\frac{1}{4} =$

$3 \times 3 = 9$

"What would be the range of answers we could get when we multiply this time?"
(Between 6 and 9)

Fraction times a whole number

$0 \times 15 = 0$

$\frac{2}{3} \times 15 =$

$1 \times 15 = 15$

"What are the limits for the answer as indicated by this table?"
(Between 0 and 15)

Fraction times a fraction

$0 \times 0 = 0$

$\frac{1}{2} \times \frac{1}{4} =$

$1 \times 1 = 1$

"When multiplying a fraction times a fraction (both proper), what would be the range of answers we might get?"
(Between 0 and 1)

Fraction times a mixed number

$0 \times 2 = 0$

$\frac{3}{4} \times 2\frac{1}{3} =$

$1 \times 3 = 3$

"Here, with this $\frac{3}{4}$ times $2\frac{1}{3}$, what would be the limits of our answer?"
(Between 0 and 3)

Mixed number times a whole number

$2 \times 3 = 6$

$2\frac{1}{4} \times 3 =$

$3 \times 3 = 9$

"With this problem what are the possibilities of your answer?"
(Between 6 and 9)

Mixed number times a fraction

$1 \times 0 = 0$

$1\frac{1}{2} \times \frac{1}{3} =$

$2 \times 1 = 2$

"What are the answer possibilities here?"
(Between 0 and 2)
"Would that be true every-time you multiply a mixed number times a fraction?"
(No, only this particular problem. Other problems will have their own limitations.)

Mixed number times a mixed number

"Here is another problem. What are the limits for this answer?" (Between 2 and 6)

$2 \times 1 = 2$

$2\frac{1}{2} \times 1\frac{1}{4} =$

$3 \times 2 = 6$

<u>The shortcut called cancellation</u>:

$\frac{4}{4} \quad \frac{7}{7} \quad \frac{10}{10} \quad \frac{100}{100}$

"Think of some fractions that equal 1 (are names for 1)."

$1 \times \frac{2}{7} = \frac{1}{1} \times \frac{2}{7} = \frac{2}{7}$

"What happens when we multiply with one; either as a whole number or written as a fraction?"

$\frac{3}{4} \times 1 = \frac{3}{4} \times \frac{1}{1} = \frac{3}{4}$

$\frac{2}{2} \times \frac{4}{9} = \frac{8}{18} = \frac{4}{9}$

$\frac{4}{5} \times \frac{3}{3} = \frac{12}{15} = \frac{4}{5}$

(We always get the other factor).

$1 \times \frac{2}{3} = \frac{2}{3}$

$\frac{4}{4} \times \frac{6}{8} = \frac{6}{8}$

"Does that mean we could just eliminate multiplying with one?
Would a sentence like this be true?"

$\frac{24}{24} \times \frac{17}{20}$

"What do we usually do first with a problem like this?" (Multiply; 24 x 17 and 24 x 20.)
"Let's do it."

$\frac{24}{24} \times \frac{17}{20} = \frac{408}{480}$

"Now what do we do?" (Divide the numerator and denominator by the largest number we can - 24.)

$\frac{24}{24} \times \frac{17}{20} = \frac{408}{480} = \frac{17}{20}$

"Let's do that."

"What was our answer?"

$\left(\frac{17}{20}\right)$

"Do you see that fraction in the original problem?" (Yes)
"What did we really multiply $\frac{17}{20}$ by?"

(One)

"And do we know that when we multiply by one we get the other factor?" (Yes)

"Then could we have eliminated all that multiplying and dividing by just recognizing the one at first?" (Yes)

$\frac{7}{8} \times \frac{8}{9}$

"Let's try a problem where the one is not so easy to find."

$\frac{7 \times 8}{8 \times 9}$

"Could we write it this way?"
(Yes)

$\frac{8 \times 7}{8 \times 9}$

"Could we change the order and still keep the same value?"
(Yes)

$\frac{8 \times 7}{8 \times 9}$

"What do you see in this problem now?"

$\left(1 \times \frac{7}{9}\right)$

"What does one times a number give us?"
(the other factor)
"What is one times $\frac{7}{9}$?"

$\left(\frac{7}{9}\right)$

"Let's check it with the longer method."

$\frac{7}{8} \times \frac{8}{9} = \frac{56}{72} = \frac{7}{9}$

"What steps did the shortcut save us?"

(Multiplying the numerators and denominators, then dividing by the greatest common factor.)

$\frac{3}{4} \times \frac{8}{11}$

"Sometimes the shortcut is not so obvious; we will have to look harder."

"We must look for common factors (or factor the numerals in the fraction) to find the equivalent of 1."

$$\frac{3 \times 8}{4 \times 11} = \frac{3 \times 2 \times 4}{4 \times 11}$$

"Do you see the one now?"

$$\frac{3 \times 2 \times \cancel{4}}{11 \times \cancel{4}}$$

"Does changing the order help you see the one?"

(Yes, $\frac{4}{4}$)

$$\frac{3 \times 2}{11} = \frac{6}{11}$$

"So what is left to do?" (Multiply 3 x 2)

$$\frac{3}{4} \times \frac{8}{11} = \frac{24}{24} = \frac{6}{11}$$

"Here is the long way. What steps did our shortcut eliminate?" (Multiplying 4 x 11 and dividing to change to simplest terms.)

$$\frac{3}{5} \times \frac{1}{3}$$

"Try this. Do you see a one? Remember, there must be a common factor in the numerator and the denominator to equal one."

(Yes, $\frac{3}{3}$)

$$\frac{\cancel{3}}{5} \times \frac{1}{\cancel{3}}$$

"If we eliminate the 1, what remains?"

($\frac{1}{5}$)

$$\frac{4}{9} \times \frac{3}{5}$$

"Let's try a harder problem where the one is not so easy to see. Remember, we are looking for common factors. Do you see a one?" (Yes, $\frac{3}{3}$ in the 3 and the 9)

"One is the other factor of 3, what is the other factor of 9?"

(3)

"Record this."

$\frac{4}{3} \times \frac{3}{5}$

"Now, when we multiply, what do we get?"

$\left(\frac{4}{15}\right)$

"Is it already in simplest terms?"
(Yes)

Note: A mixed number must be written as an improper fraction before the shortcut can be used.

"Sometimes we can find more than one fraction equivalent to one, especially when we multiply fraction x fraction x fraction.

$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$

"Do you see a one?"

$\left(\text{Yes } \frac{3}{3}\right)$

$\frac{2}{1} \times \frac{3}{4} \times \frac{4}{5}$

"Eliminate it. Do you see another one?"

$\left(\text{Yes } \frac{4}{4}\right)$

$\frac{2}{3_1} \times \frac{3'}{4_1} \times \frac{4'}{5}$

"Eliminate this."
(One may or may not be recorded as a factor. If the factor is not "one" it must be recorded.

"Can you see the answer without any computational work?"

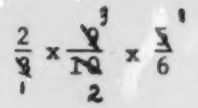
(Yes, $\frac{2}{5}$)

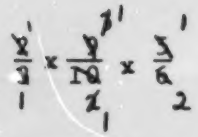
"By using the shortcut once we may see a chance to use it again in the same problem."

$\frac{2}{3} \times \frac{9}{10} \times \frac{5}{6}$

"Do you see any ones?"

(Yes, $\frac{3}{3}$ and $\frac{5}{5}$)





"Now do you see any more ones?"

Yes, $\frac{2}{2}$ and $\frac{3}{3}$

"What's left?"

$\frac{1}{2}$

"What's the answer?"

$\frac{1}{2}$

Note: When using the shortcut called cancellation, especially when more than one step is involved, you might want to factor out (rename) a problem or two so the students can see that is what they are really doing.

$\frac{2}{3} \times \frac{9}{10} \times \frac{5}{6}$ is $\frac{2 \times 9 \times 5}{3 \times 10 \times 6}$

becomes

$$\frac{2 \times 3 \times 3 \times 5}{3 \times 2 \times 5 \times 2 \times 3}$$

These factors can be rearranged if necessary to see the ones clearly.

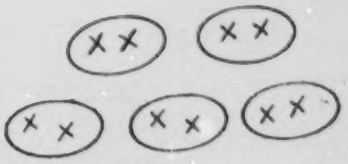
$$\frac{2 \times 3 \times 5 \times 3}{3 \times 2 \times 5 \times 2 \times 3}$$

$$\frac{2 \times 3 \times 3}{3 \times 2 \times 2 \times 3}$$

$$\frac{3 \times 3}{3 \times 2 \times 3}$$

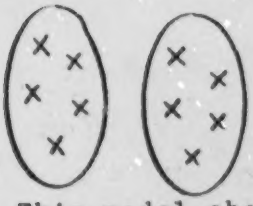
Division
Meaning of Division
(whole numbers):

Note: Division sentences have two interpretations which grow out of the physical situations. (Refer to pages 2 - 9 in Division Strategies.)



$10 \div 2 = 5$

This model shows one interpretation. 10 objects are separated, putting 2 objects in each set. Question asks how many sets of 2 objects. The corresponding multiplication equation is $n \times 2 = 10$



This model shows the other interpretation. 10 objects are separated into 2 sets. Question asks how many objects in each set. The corresponding multiplication equation is $2 \times n = 10$.

Both interpretations would also apply to fractional numbers, but the second one is very difficult to model. The committee suggests that teachers make initial presentations with the first meaning.

All work should be examined for reasonableness of results. Help students develop estimation skills.

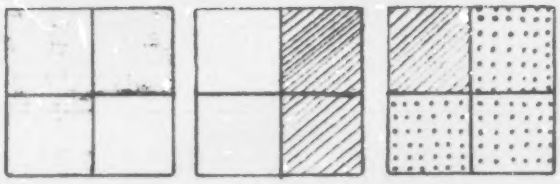
"I want to find the number of sets of 3 I can make from this set of 15 blocks. What division sentence can we write?" ($15 \div 3 = n$)
"Yes, we use division to find the number of sets in the whole collection."

"What division sentence would we write to find the number of one-halves in these 3 circles?"

$\left(3 \div \frac{1}{2} = \square\right)$

"Let's cut the circles to find the answer. What is it?" (6)

"Is that answer reasonable?"
(Yes) "Did you notice that you divided by a fraction and got an answer which is greater than the dividend?"



"Try this one."

$3 \div \frac{3}{4} = \square$

Color $\frac{3}{4}$ red, $\frac{3}{4}$ blue, $\frac{3}{4}$ yellow, and $\frac{3}{4}$ green."

There are 4 sets of $\frac{3}{4}$ in 3.
"So $3 \div \frac{3}{4} = 4$. Is the answer greater than 3?"
(Yes)
"Is it reasonable?"
(Yes)

"What question does this sentence ask?
$\frac{1}{3} \div \frac{2}{3} = \square$ "
(How many $\frac{2}{3}$'s are in $\frac{1}{3}$.)

"Let's lay $\frac{2}{3}$ on top of $\frac{1}{3}$."

"Does $\frac{2}{3}$ fit on $\frac{1}{3}$?" (No)

"What fraction, or part, of $\frac{2}{3}$ is on $\frac{1}{3}$?" $\left(\frac{1}{2}\right)$

Note: Transparent fraction circles are useful for this measurement.

"So $\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$. Is the answer greater than the dividend?"

Note: Teacher should be alert to the student's thinking at this point. Be certain that he/she knows that the $\frac{1}{2}$ means $\frac{1}{2}$ of the $\frac{2}{3}$, not $\frac{1}{2}$ of 1. The mental picture should not be:

F-49 **Reciprocals:**

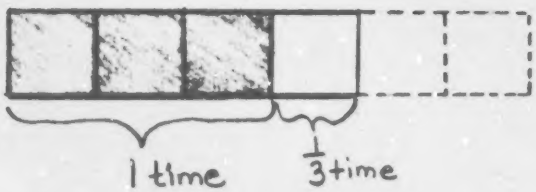
$1 \div \frac{1}{2} = 2$

"When we divide 1 by a fraction, we get a special kind of number. Let's see what happens. One divided by $\frac{1}{2}$ equals what?" (2)
"Yes, and we can write that $\frac{2}{1}$."

Teacher write:

$1 \div \frac{1}{2} = \frac{2}{1}$

(This begins a table.)



"How about $1 \div \frac{3}{4}$? How many times will $\frac{3}{4}$ fit on 1?" (1 time and $\frac{1}{3}$ of another time)
"So $1 \div \frac{3}{4} = 1\frac{1}{3}$, or $\frac{4}{3}$."

Teacher adds to table:
$1 \div \frac{3}{4} = \frac{4}{3}$

$\frac{5}{8} \times \frac{8}{5} =$

$\frac{3}{4} \times \frac{4}{3} =$

$\frac{12}{7} \times \frac{7}{12} =$

$1\frac{1}{5} \times \frac{5}{6} =$

$6 \times \frac{1}{6} =$

$\frac{4}{5} \times \frac{\square}{\square} = 1$

$\frac{\square}{\square} \times \frac{2}{3} = 1$

$1\frac{2}{5} \times \frac{\square}{\square} = 1$

$\frac{\square}{\square} \times 3 = 1$

$1 \div \frac{4}{5} = \frac{\square}{\square}$

$1 \div \frac{8}{1} = \frac{\square}{\square}$

$1 \div 1\frac{3}{4} = \frac{\square}{\square}$

"Here are some problems to solve."

"These pairs of numbers are reciprocals:

$\frac{2}{3}$, $\frac{3}{2}$

$\frac{1}{4}$, $\frac{4}{1}$ or 4

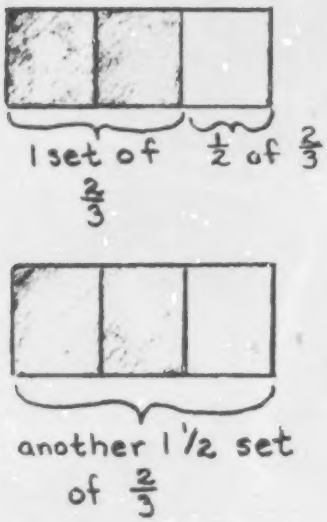
Can you make up some more pairs?"

"Two fractional numbers are reciprocals if their product is 1. When 1 is divided by a fraction, the missing factor is the reciprocal of the fraction.

$1 \div \frac{5}{6} = \frac{\square}{\square}$

Think: $\frac{5}{6} \times \frac{\square}{\square} = 1$".

F-50 **Divide whole numbers by fractions:**



"$1 \div \frac{2}{3} = \frac{3}{2}$ or $1\frac{1}{2}$.

What would the solution for this problem $\left(2 \div \frac{2}{3}\right)$ be? Let's make a picture."

"If $1 \div \frac{2}{3} = \frac{3}{2}$, then $2 \div \frac{2}{3}$ would be twice as much.

Let's start a table."

$1 \div \frac{2}{3} = \frac{3}{2}$

$2 \div \frac{2}{3} = 2 \times \frac{3}{2} = \frac{6}{2}$

"Would $3 \div \frac{2}{3}$ be $3 \times \frac{3}{2}$?"
(Yes)
"Let's continue the table."

"Does it look like we've changed the division problem to a multiplication problem? What number have we multiplied by?" (the reciprocal of the divisor.)

Lead students by modeling and questioning to develop the rule: To divide by a fraction, multiply the dividend by the reciprocal of the divisor.

Note: An alternate explanation for the "invert and multiply" rule uses mathematical logic. Some students may be able to use it.

$\frac{5}{6} \div \frac{3}{4}$ can be written as a fraction because a fraction means division. (In this guide we have not developed the idea that a fraction means division... $\frac{17}{4} = 17 \div 4$)

$$\frac{5}{6} \div \frac{3}{4} = \frac{\frac{5}{6}}{\frac{3}{4}}$$

To simplify this fraction, we decide to change the denominator to 1. If we multiply $\frac{3}{4}$ by its reciprocal, the denominator will be 1. The value of the whole expression

$$\frac{\frac{5}{6}}{\frac{3}{4}}$$

will be maintained if we multiply both the numerator $\frac{5}{6}$ and the denominator $\frac{3}{4}$ by the same number.

$$\frac{\frac{5}{6} \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}} = \frac{\frac{5}{6} \times \frac{4}{3}}{1}$$

F-50 Divide any fraction,
F-51 whole number, or
F-52 mixed number by
F-53 any fraction, whole
F-54 number or mixed
F-55 number:

$\frac{2}{3} \div \frac{3}{8}$

"1 divided by $\frac{3}{8}$ would be what number?" $\left(\frac{8}{3}\right)$

"Would $\frac{2}{3} \div \frac{3}{8}$ be as much as that?" (No)
"Would it be $\frac{2}{3}$ as much?" (Yes)

"So $\frac{2}{3} \div \frac{3}{8} = \frac{2}{3} \times \frac{8}{3}$.

The same rule applies to this kind of problem. To divide a whole number by a fraction we multiply by the reciprocal of the divisor. We do the same thing when we divide a fraction by a fraction.

Remember what we did with multiplication of fractions? That is, we changed all whole numbers and mixed numbers to improper fractions, then treated them like proper fractions.

Here are some sample problems of each kind."

whole number ÷ fraction

$6 \div \frac{7}{8} = \square$

fraction ÷ fraction

$\frac{5}{6} \div \frac{1}{3} = \square$

mixed number ÷ fraction

$2\frac{1}{2} \div \frac{3}{4} = \square$

whole number ÷ mixed number

$9 \div 2\frac{2}{3} = \square$

mixed number ÷ whole number

$6\frac{3}{4} \div 2 = \square$

mixed number ÷ mixed number

$6\frac{3}{5} \div 2\frac{1}{5} = \square$

Note: Many computational errors will be prevented if students can be convinced that they should not do too many computations in one step.

First, change all whole numbers and mixed numbers to fractional form.

$6\frac{3}{5} \div 1\frac{1}{10} = \frac{33}{5} \div \frac{11}{10}$

Second, use the invert and multiply rule.

$\frac{33}{5} \times \frac{10}{11}$

Third, simplify by "cancellation" if desired.

$\frac{\cancel{33}^{3}}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{\cancel{11}_{1}}$

Fourth, compute and simplify the product.

$\frac{3}{1} \times \frac{2}{1} = \frac{6}{1} = 6$

DECIMAL FRACTIONS

DECIMALS

The teaching of decimals can be approached in several ways. One is to build on a child's prior knowledge of fractions to develop skill in decimal concepts and the four basic operations. However, by using this method it is assumed that a child is very adept with fractions.

This guide more often looks on decimals as an extension of the numeration system. The same relationship exists between place value positions to the right of the decimal point as exists to the left of the decimal point.

To introduce the numeration system to the right of the decimal point, modeling is the best approach. Metric/Dienes blocks are a preferred modeling tool. Graph paper and money are alternate ways to create models. At some point no model will be needed and the child will work at an abstract level. It is possible to teach decimal operations by modeling but this often becomes contrived. It is easier to parallel the similarities between decimal numbers and whole numbers in the four basic operations.

This guide also puts great emphasis on estimation to promote "thinking" computation rather than rote computation. By estimating, the student verifies algorithms he has discovered.

This guide is a <u>suggested</u> sequence of steps for teaching decimals. The activities are a sample of what could be done. Many more activities would be needed at each step.

The teacher should <u>guide</u> the student through the activities. This does not mean "telling" but means structuring the situation so the child can discover for himself while the teacher guides through questioning.

The Performance Objective numbers are listed to the left of some objectives and activities. The numbers indicate that decimals are introduced at level E (fifth grade) but a teacher could introduce decimals in fourth grade if it is felt the students are ready. All the Performance Objectives are meant to be flexible guides.

DECIMALS

The initial step in teaching decimals is to develop the decimal concept which is to recognize and understand:

(1) that a decimal fraction, less than one, is a part of a whole.

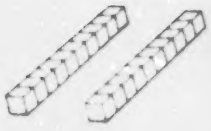
(2) that a relationship exists between fractional parts to each other and to one whole. This allows application of trading (regrouping) concepts.

E-1 Objective: Develop and verbalize the decimal concept including its written form.

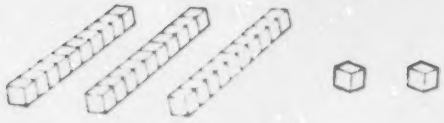
Activity: Use Dienes/metric blocks with the cube representing one.

1.

1. "If the cube is one, what is represented here? (3) Record that number."

2. 

2. "Record the number represented by the bars." (20)

3. 

3. "Record the number shown by the cubes and bars." (32)

Activity: Use Dienes/metric blocks with the bar representing one.

1.

1. "Suppose the bar is one; what is represented here?" (4)

2.

2. "What is represented here?" (1)

3.

3. "Since the bar is one, what part of the bar is one cube?" (one tenth)"
"How can we record that?" $\left(\frac{1}{10}\right)$
"We have recorded this as a fraction. We can also record this in a new way called a decimal which is written .1 . In writing a decimal, a decimal point is used to separate the whole numbers from the parts of the whole. The whole numbers are to the left of the decimal point and the parts are to the right."

4.

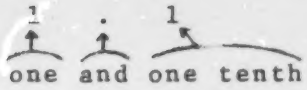
4. "Since the bar is one what is represented here?" (two tenths)
"How would you record this in decimal form?" (.2)

Note: Continue examples like the above.

5.

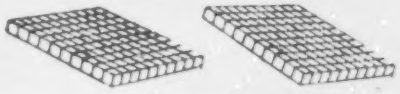
5. "What is represented here?" (one and one tenth) "In decimal form this is written 1.1 ."

Note: Please stress the correlation between the word form and the decimal form.

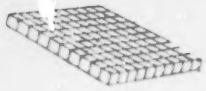


E-2
E-3
E-4
E-5
E-8

Activity: Use Dienes/metric blocks with the flat representing one.

1. 

1. "Suppose the flat is one, what is represented here?" (2)

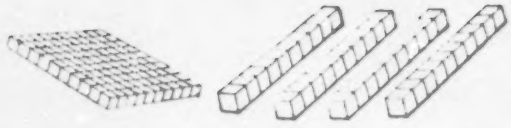
2. 

2. "What is represented here?" (1)

3.

3. "If the flat is one, what does this one bar represent?" (.1)

Alternate: "What part of the flat is the bar?" (.1)

4. 

4. "What is represented here?" (1.4)

5.

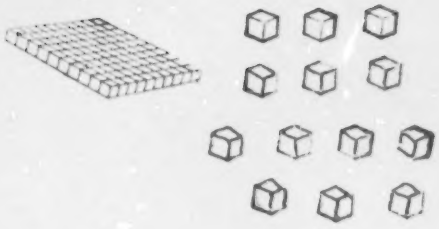
5. "How many cubes in a flat?" (100) "What does this one cube represent?" (one hundredth)

"How can we record that?"
$\left(\frac{1}{100}\right)$
"We can also record this as a decimal which is written .01."

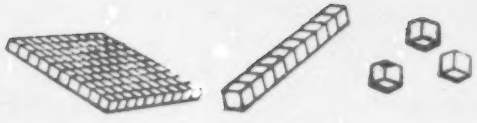
Note: The decimal system is an extension of the place value system. The first place to the right of the decimal point is the tenths column, the second place to the right of the decimal point is the hundredths column.

6.

6. "What is represented here?" (1.06)

7. 

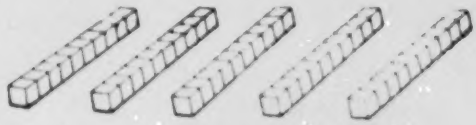
7. "What is represented here?" (1.13) "Is there another way to model by doing some trading? Let's picture it."



"To make 1.13 easier to count we trade 10 hundredth cubes for 1 tenth bar but continue to read it as one and 13 hundredths."

Note: When teaching the decimal concept through modeling, urge the student to model for himself. Read and record decimals using concept terminology and format.

E6 Activity: Use Dienes/metric blocks to show the relationship that exists between various decimal fractions. (ordering)

1. 

1. "If the flat is one, what is represented here?" (.5)

2.

2. "What is represented here?" (.23)

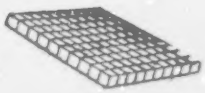
3. "Compare the two models of .5 and .23. Which is larger/smaller?" (.5)
Alternate: "Which has the greater/lesser value?"

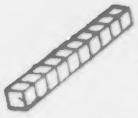
Note: Continue with many examples especially ordering fractions when one is expressed as tenths and the other as hundredths.

Activity: Reinforce the decimal concept with money using dollars (ones) dimes (tenths) and pennies (hundredths) to equate money with decimals.

Note: Assuming the students have a strong understanding of money, it can be used as a means of extending and enriching the child's grasp of decimals but not as material for introducing the concepts.

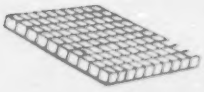
1. Give student these materials or use for demonstration with the flat representing one.





1. Review relationships of metric blocks including numerical representation.

Expected response:



1.

.1

.01

2. Give student these materials for use or demonstration.

$1.00

2. Review money value used and their relationships including written form.

Expected response:

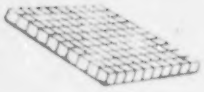
$1.00

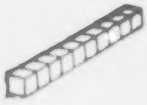
$1.00

$ .10

$ .01

3. Give students these materials or use for demonstration.

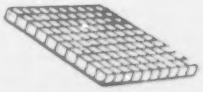




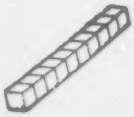
$1.00

3. "How do you pair one metric block with the money given?"

Expected sclution:



1. $1.00 $1.00



.1 $ .10

.01 $ .01

Note: Point out difference in written format.

4. Alternate money and decimal problems to reinforce comparison.

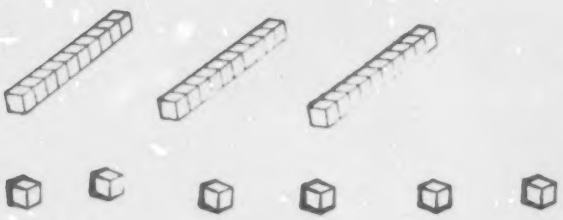
Note: Give students dimes and pennies only.

4. Reinforce comparison by asking such things as "Using metric blocks and money show .36 and $.36."

Expected solution:

$.36

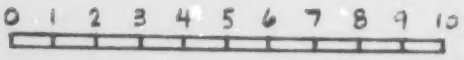
Expected solution:



.36

Objective: Apply decimal concepts to measurement (length and area)

Activity: Use meter stick to relate decimals to length.

1. 

Establish and label meter as one. Place 10 bars along meter stick.
(one bar = one decimeter in length. Do not call it a decimeter at this time.)

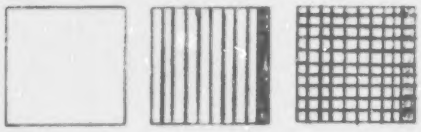
"If the meter is one, what is one bar?"
Correct response: .1 meter
"What is two bars?" Etc.
Record each response.

2. Place 10 cubes on one bar along meter stick.
"If the meter is one and the bar is .1 meter, what is 1 cube?" (.01)
Record response for each cube added.

Activity: Use number line to relate decimals to length.

Number lines of varying lengths may be used; however, be sure each line is divided into ten equal parts.

Activity: Use metric blocks (Dienes) to show congruent regions.



"If the flat is one, what is one bar?"
Response: .1 flat
"Since this flat measures 100 cubes, what do 10 cubes measure?"
Response: .10 flat
"What do you notice about the size of these two shaded regions?"
Response: same
"Therefore, we can conclude that .1 = .10."

Continue activity with:
.2 = .20
.5 = .50
etc.

Activity: Use meter sticks and/or number line to show equivalent decimals.

Meter stick or number line

.1 meter = .10 meter

"If this meter is one, what is one bar?" .1 meter

"Since this meter measures 100 cubes, what do 10 cubes measure?" .10 meter

"If we slide the bar representing .1 meter on top of the 10 cubes representing .10 meter, what do you notice?"
Response: same

Continue activity with:
.2 m = .20 m
.5 m = .50 m
etc.

"Therefore, we can conclude that .1 = .10 "

NOTE: Mathematically .1 and .10 are equivalent but when used in measurement they are no longer exactly equivalent. .1 meter is a less precise measurement than .10 meter . Do not stress this with students.

E-10
E-12

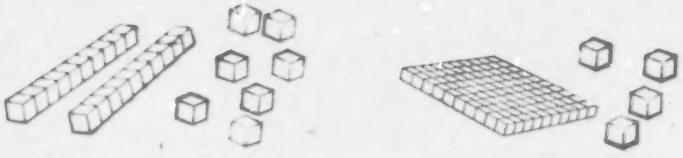
Objective: Add and subtract decimals.

Activity: Use metric blocks or other manipulatives to model the process of adding hundredths to hundredths, tenths to tenths, ones to ones, tens to tens, etc.

Example:

43.2	46.8
+26.5	+27.5
57.9	43.5
-35.6	-26.6
$36.17	$55.78
+$41.82	+$42.76

If = 1



.27 + 1.04

Review with students that 10 tenths is one and 10 one-hundredths is one tenth. You may review trading (regrouping) with metric blocks. (Refer to addition and subtraction strands of this guide.)

Activity: Add and subtract decimals when adding or subtracting tenths and hundredths.

Note: When addends are expressed in tenths and hundredths, students must use prior knowledge of equivalent decimals.

Example:

$$\begin{array}{r} 36.4 \\ +27.54 \\ \hline \end{array} = \begin{array}{r} 36.40 \\ +27.54 \\ \hline \end{array}$$

The same is true in subtraction.

Example:

$$\begin{array}{r} 38.5 \\ -29.37 \\ \hline \end{array} = \begin{array}{r} 38.50 \\ -29.37 \\ \hline \end{array}$$

Note: Include examples written in horizontal form to stress the necessity of adding tenths to tenths, hundredths to hundredths, etc.

The student should eventually conclude when adding like values that the decimal points do "line up".

E-15
E-16
E-17
E-18

Objective:
Multiply decimals

Activity: Review approximating each decimal to the nearest whole number.

As preparation for estimating products:

(a) list various decimals, asking child if greater than or less than one.

Examples:

$$\begin{array}{rcl} .2 & < & 1 \\ 4.6 & > & 1 \\ .68 & < & 1 \\ 4.03 & > & 1 \end{array}$$

(b) list various decimals, asking child which two whole numbers the decimal is between and closer to (rounding).

Examples:

$$\begin{array}{ccc} \underline{8} & 8.2 & \underline{9} \\ \underline{8} & 8.69 & \underline{9} \\ \underline{0} & .7 & \underline{1} \end{array}$$

Activity: Multiply decimals by rounding and estimating.

Examples: Estimate the following products:

6 x 4 = 24 (exact)

6.2 x 4 $\approx$ 24 (* footnote)

5.8 x 4 $\approx$ 24

7 x 5 = 35 (exact)

7.1 x 5.3 $\approx$ 35

6.9 x 4.8 $\approx$ 35

Proceed with examples to the hundredths place. Include problems written in vertical form.

Activity: Compute exactly, placing decimal using estimated product.

Use examples above making comparison.

$$\begin{array}{r} 6.2 \\ \times\ \ 4 \\ \hline 248 \end{array}$$

"Use your estimated product to determine where to place the decimal point. We know from estimating our product is close to 24. Using that information, would our product be 248, 24.8, 2.48 or .248?"
Correct response is 24.8 which is closest to 24.

Activity: Compute exactly using algorithm.

"Many examples like the above should be done before noting the algorithm for placing the decimal in the product.
"What rule have you discovered when multiplying decimals?"
(After multiplying the two factors, count the total number of numerals to the right of the decimal point in each factor. The product should have the same number of numerals to the right of the decimal point).

* $\approx$ This sign represents "is approximately".

Activity:
Multiply decimals
using fractions.

NOTE: You can use a student's prior knowledge of fractions in teaching the multiplication of decimals. Be cautious in using this method because the student needs to be very proficient with fractions and renaming them in lowest terms.

Activity: Rewrite decimal fractions less than one as proper fractions, those greater than one as mixed numbers, then as improper fractions. Now multiply the fractions.

Use this as a way to justify the usual algorithm. This could be used briefly to reconvince students of the correctness of the procedure concerning the placement of the decimal point.

Examples: .6 x .7 can be written

$$\frac{6}{10} \times \frac{7}{10} = \frac{42}{100} = .42$$

$$6.2 \times 4 = 6\frac{2}{10} \times 4 =$$

$$\frac{62}{10} \times \frac{4}{1} = \frac{248}{10} = 24.8$$

F-26
F-27
F-28
F-29

Objective: Divide decimal number by whole number.

Activity: Do several easy division problems using the missing factor method.

Examples:

$2\overline{).8}$ 2 x ____ = .8 $4\overline{).32}$ 4 x ____ = .32

$3\overline{).9}$ $9\overline{).81}$

$4\overline{).4}$ $6\overline{).66}$

Discover that tenths divided by a whole number is tenths and hundredths divided by a whole number is hundredths.

Activity: Record whole numbers divided by a whole number and decimal divided by a whole number to create a pattern.

$2\overline{)8000}$ or $2 \times ____ = 8000$

$2\overline{)800}$ or $2 \times ____ = 800$

$2\overline{)80}$ or $2 \times ____ = 80$

$2\overline{)8}$ or $2 \times ____ = 8$

$2\overline{).8}$ or $2 \times ____ = .8$

$2\overline{).08}$ or $2 \times ____ = .08$

$2\overline{).008}$ or $2 \times ____ = .008$

After developing patterns, notice that when dividing by a whole number the decimal point in the quotient is directly above the decimal point in the dividend.

F-27
F-28
F-29

Objective: Dividing Decimals by Decimals

It should already be established through prior fraction work that $\frac{2}{3}$ can be read "2 divided by 3".

Activity: Show that when multiplying the dividend and divisor by powers of 10, the missing factor (quotient) remains the same.

To support the concept in the stated activity, review fractions and multiplying by the identity "one" (as powers of 10).

Examples:

(a) $\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{2 \times 100}{3 \times 100}$

(b) $\frac{.6}{.2} = \frac{.6 \times 10}{.2 \times 10} = \frac{6}{2}$

(c) $.2\overline{).6} \rightarrow (.2 \times 10)\overline{)(.6 \times 10)} \rightarrow 2\overline{)\,^{3}6}$

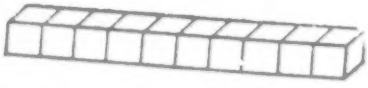
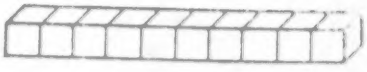
$.02\overline{).06} \rightarrow (.02 \times 100)\overline{)(.06 \times 100)} \rightarrow 2\overline{)\,^{3}6}$

Because it is simpler to divide by a whole number, multiply the decimal divisor by the appropriate power of 10 to rename it as a whole number. The dividend then must be multiplied by the same power of 10.

Note: In all work with dividing decimals, encourage checking.

Examples: $.2\overline{)\,^{3}.6} \rightarrow .2 \times 3 = .6$

$.02\overline{)\,^{3}.06} \rightarrow .02 \times 3 = .06$

Activity:
Computation

The division and the checking process are the same as with whole numbers.